Belham
Coll. Regal. 1730

Euclid

THE ELEMENTS OF EUCLID Explain'd,

in a *new*, but most *easie* method,

Together with
The Use of every Proposition through all parts of the Mathematicks.

Written in *French*, by that Excellent Mathematician,

F. CLAUD. FRANCIS MILLIET de CHALES,

of the *Society of JESUS*.

And now carefully done into *English*, and purg'd from a multitude of Errors, which had escap'd in the Original.

OXFORD,
Printed by *L. Lichfield*, Printer to the University, for *Anthony Stephens*, Bookseller near the *Theater* in *Oxford*. 1685.

THE PREFACE.

HAving for a long time obſerv'd, that moſt of thoſe, that take in hand the Elements of Euclid, are apt to diſlike them, becauſe they cannot preſently diſcern, to what end thoſe ſeemingly inconſiderable, and yet difficult Propoſitions, can conduce: I thought I ſhould do an acceptable piece of ſervice, in not only rendring them as eaſie as poſſible, but alſo adding to each Propoſition a brief account of ſome Uſe, that is made of them in the other parts of the Mathematicks. In proſecuting which deſign, I have been oblig'd to change ſome Demonſtrations, that ſeem'd too intricate and perplex'd, and above the ordinary capacity of Beginners, and to ſubſtitute others more intelligible in their ſtead. For the ſame reaſon, I have demonſtrated the fifth Book after a method, much more clear, than that by Equimultiples, formerly uſed. I would nat be thought to have ſet down all the Uſes, that may be

made of these Propositions: to have done that, would have oblig'd me to have compris'd the whole Mathematicks *in this one Book; which would have render'd it both too large, and too difficult. But I have contented my self with the choice of such, as may serve to point out some of the Advantages they afford us, and are also in themselves most clear, and most easie to be apprehended. I have distinguish'd them by* * Inverted Commas, *that the Reader may know them; not desiring he should dwell too long upon them, or labour to understand them perfectly at first, since they depend on the Principles of the other Parts.*

This therefore being the design of this small Treatise, I voluntary offer it to the publick, in an Age, whose Genius *seems more addicted to the* Mathematicks, *than any that has preceded it.*

* Instead of the Authors Italian character.

Eight

Eight Books of the Elements *of* EUCLID, *together with the Use of the* Propositions.

THE FIRST BOOK.

' THE design of *EUCLID* in this Book
' is to lay down the First Principles of
' *Geometry*; and to do it methodically,
' he begins with *Definitions*, and the expli-
' cation of the most ordinary Terms. To these
' he adds some *Postulata*; and then propo-
' sing those known *Maxims*, in which natu-
' ral reason does instruct us, he pretends, not to
' advance a step farther without a *Demonstration*,
' but to convince every man, even the most ob-
' stinate, that will grant nothing, but what is ex-
' torted from him. In the first Proposition, he
' treats of *Lines*, and the different *Angles*, which
' are

'are form'd by their concourſe; and having oc-
'caſion to compare divers *Triangles* together,
'in order to demonſtrate the Properties of
'*Angles*, he makes that the buſineſs of the
'Eight firſt Propoſitions. Then follow ſome
'Practical Inſtructions, how to divide an *Angle*
'and a *Line* into two equal parts, and to draw a
'*Perpendicular*. Next he ſhows the properties
'of a *Triangle*, together with thoſe of *Parallel*
'*Lines*; and having thus finiſh'd the Explica-
'tion of this firſt figure, he paſſes on to *Paral-*
'*lelograms*, teaching the manner of reducing
'any *Polygone*, or multangular figure into one
'more regular. Laſtly, he finiſhes the firſt
'Book with that famous Propoſition of *Pytha-*
'*goras*, That *in every rectangular Triangle the Square of the* * *Baſe is equal to the Squares of both the other Sides*.'

* He calls that the *Baſe*, which is commonly call'd the *Hypotenuſe*, i.e. the Line that is oppoſite to the right Angle.

DEFINITIONS.

1. A *Point is that which hath no parts*.

'This Definition muſt be underſtood in this
'ſenſe: That quantity, which we conceive
'without diſtinguiſhing its parts, or ſo much as
'conſidering whether or no it has any, is a *Ma-*
'*thematical* point; which is therefore very

'dif-

'different from those of *Zeno*, which were sup-'pos'd to be absolutely indivisible, and there-'fore such, that we may reasonably doubt whe-'ther they are possible; but the former we can-'not doubt of, if we conceive them aright.

2. *A Line is length without breadth.*

'The sense of this definition is the same 'with the former, That quantity, which we 'conceive as length, without reflecting on its 'breadth or thickness, is that, which we under-'stand by a Line; though it be impossible to 'draw a real Line, which will not be of a cer-'tain breadth. 'Tis commonly said, that a Line 'is produc'd by the motion of a Point; which 'ought to be carefully observ'd; for motion 'may on that manner produce any quantity 'whatsoever: but here, we must imagine a 'Point to be only so mov'd, as to leave one 'trace in the space, through which it passes, and 'then, that trace will be a line.

3, *The two Extreams of a Line are Points.*

4. *A right Line is that, whose points are equally plac'd between the two Extreams.*

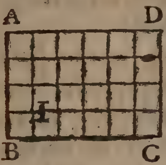
'Or thus. A right Line is the shortest that 'can be drawn from one point to another. Or 'yet. The Extreams of a right Line may cast a 'shadow upon the whole Line.

5. *A Superficies or Surface is a quantity, to which is attributed length, and breadth, without the consideration of any thickness.*

6. The

6. *The Extreams of a Superficies are Lines.*

7. *A plane or right Superficies is that, whose lines are equally plac'd between its two Extreams; Or that, to which a right line may be every way apply'd.*

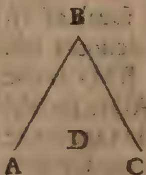


'I have before observ'd, that 'motion may produce any quan'tity whatsoever: accordingly 'we say, when one line moves 'over another, it produces a Su'perficies, or a Plane; and that 'that motion has a kind of affinity with *Arith'metical* Multiplication. Suppose then the 'line AB to pass along the line BC, retaining 'still the same situation, without any inclination 'to one side or other: the point A will describe 'the line AD, the point B the line BC, and 'the intermediate points the lines parallel to 'those, which will make up the Superficies A'BCD. I add further, that this motion answers 'to *Arithmetical* Multiplication; because did 'I know the number of points, that are contain'd 'in both those lines, AB, and DC; by multi'plying them together, I should find a product, 'which would give me the number of points, 'which constitute the whole superficies ABCD. 'As for example, if AB contain'd four points, 'and BC six, by saying four times six make 'twenty four, I find, that the whole superficies 'ABCD consists of twenty four points. Now by

‘ by a *Mathematical* point, may be underſtood ‘ any quantity whatſoever; *e.g.* a Foot, provi-‘ ded it be not ſubdivided into parts.

8. *A plain Angle is the * diſtance or opening of two lines touching each other, ſo as not to compoſe only one line.*

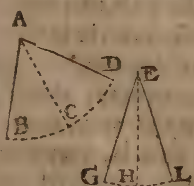
[* *Overture.* Gall. πρὸς ἀλλήλας κλίσις. Eucl.]



‘ As the diſtance D betwixt the ‘ lines AB, and BC; which are not ‘ parts of the ſame line.

9. *A Rectilineal Angle is the diſtance betwixt two right lines.*

‘ Tis chiefly of this ſort of An-‘ gles that I would be underſtood at preſent; ‘ which I define by *diſtance or opening*, becauſe ‘ Experience teaches, that the greateſt part of ‘ Beginners deceive themſelves in meaſuring ‘ the greatneſs of an Angle by that of the lines, ‘ within which it is contain'd.



‘ The Angle that is more ‘ open, is the greater; that ‘ is, when the lines of one an-‘ gle lie more apart from each ‘ other than thoſe of another, ‘ taking them at the ſame di-‘ ſtance from the points of concourſe, the for-‘ mer is greater than the latter. Accordingly ‘ the angle A is greater than the angle E; be-

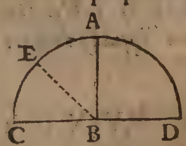
'cauſe taking the points D and B as remote 'from the point A, as the points G and L are 'from the point E; the points D and B lie far-'ther apart from each other, than the points G 'and L: from whence I infer, that if the lines 'EG and EL were produc'd farther, the angle 'E would be always of the ſame largeneſs, and 'always leſs than the angle A.

'We uſe three letters when we ſpeak of an 'Angle, of which the middlemoſt denotes the 'point of concourſe; as the angle BAD is the 'angle which by the lines BA and AD is form'd 'at the point A: the angle BAC is that made 'by the lines BA and AC: the angle CAD is 'compris'd by the lines CA and AD.

'A Circle is the meaſure of an Angle. There-'fore to know the magnitude of the Angle 'BAD, I place the foot of the Compaſs upon 'the point A, and deſcribe the circle BCD: 'the angle is ſo much the greater, by how 'many more parts of a circle the arch, that mea-'ſures it, contains: and becauſe a circle is uſu-'ally divided into 360 parts, or degrees, there-'fore an angle is ſaid to have twenty, thirty, 'forty degrees, according as the arch, com-'pris'd betwixt the lines that form it, contains 'ſo many. So the angle is the greater, which 'contains more degrees, as the angle BAD is 'greater than the angle GEL. 'The line CA 'divides the angle BAD in the middle, becauſe the

'the arches BC and CD are equal: and the an-
'gle BAC is part of the angle BAD, because
'the arch BC is part of the arch BD.

10. *When one line falling upon another makes two equal angles, they are both right angles; and the line perpendicular.*



'As for example: if the line 'AB, plac'd upon the line CD, make the angles ABC and ABD equal; that is, if, having describ'd a semicircle 'CAD from the center B, the arches AC and 'AD are equal: the angles ABC, and ABD 'are call'd right angles, and the line AB per-'pendicular. Therefore because the arch CAD 'is a semicircle, the arches CA and AD are 'each of them a quarter of a circle, that is the 'fourth part of three hundred and sixty degrees, 'that is ninety.

11. *An Obtuse angle is that which is greater than a right one.*

'As the angle EBD is an obtuse or blunt an-'gle, because its arch EAD contains more 'than a quarter of a circle.

12. *An Acute angle is that which is less than a right one.*

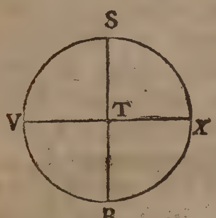
'As the angle EBC is an acute, because the 'arch EC, which measures it, has less than 'ninety degrees.

13. *A Term is the extremity or end of any quantity.*

14. *A Figure is a quantity comprehended by one or more Terms.*

'That which is call'd a Figure ought to be 'limited and inclos'd on every side.

15. *A Circle is a plain figure, terminated by the encompassing of one line, which is call'd the Circumference; and is every where equally remote from the middle point.*



'The Figure RVSX is a 'Circle, because all the 'lines TR, TV, TS, TX, 'drawn from the point T 'to the line RVSX, are e-'qual.

16. *The middle point is call'd the Center.*

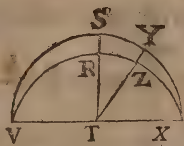
17. *The Diameter of a Circle is any line passing through the Center, and terminated at the Circumference, dividing the Circle into two equal parts.*

'As the lines VTX, and RTS.

'But if any should doubt, whether the line 'VTX does indeed divide the circle into two 'equal parts, so that the part VSX be equal to 'the part VRX; it may on this manner be 'prov'd.

'Suppose the part VRX to be plac'd upon the 'other VSX: I say, they will not exceed one

'the

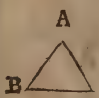


‘ the other. For if one suppose ‘ VSX exceed the other VRX, ‘ the line TR will be less than ‘ TS; and in like manner TZ ‘ than TY, which is contrary to ‘ the definition of a Circle, which affirms all ‘ the lines drawn from the center to the circum- ‘ ference to be equal.

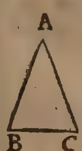
18. *A Semicircle is a figure terminated by the Diameter, and half the Circumference.*

19. *Rectilineal figures are such as are terminated by right lines, having three, or four, or five, or as many sides as you please.*

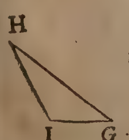
‘ *Euclid* divides Triangles with respect ei- ‘ ther to their angles, or sides.



20. *An Equilateral Triangle is that which has its three sides equal: ABC.*

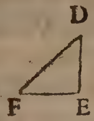


21. *An* Isosceles, *or equicrural Triangle, is that which has two sides equal:* ‘ As if the two sides AB, and AC be ‘ equal, ‘ the triangle ABC is an Iso- ‘ sceles.

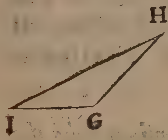


22. *A* Scalenum *is a triangle having all the three sides unequal, as GHI.*

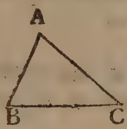
23. *A*



23. *A Rectangle triangle is that which has one right angle.*
'As DEF, supposing the angle E to 'be a right one.



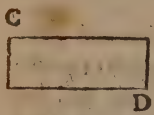
24. *An* Ambligone, *or Obtusangle triangle is that which has one angle obtuse.* As IGH.



25. Oxygone, *or Acutangle triangle is that whose angles are all acute.* As ABC.

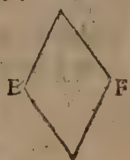
26. *A Rectangle* (properly so call'd) *is a figure consisting of four sides, and having all its angles right.*

27. *A Square has all its sides equal, and its angles right, as* AB.

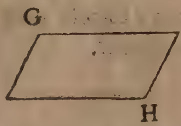


28. *An Oblong Rectangle has its sides unequal, but its angles right: as* CD.

29. *A* Rhombus, *or Losange, has equal sides, but unequal angles: as* E F.

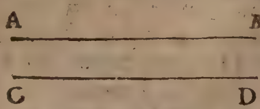


30. *A*

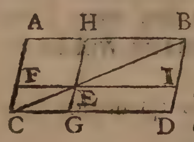


30. *A* Rhomboides, *or oblong Losange, hath both its sides and angles unequal: as* GH.

31. *Other irregular figures of four sides are call'd* Trapesia.



32. *Parallel lines are such, as being in the same plane will never concur, keeping still an equal distance one from the other: as AB, CD.*



33. *A Parallelogram is a figure, whose two opposite sides are Parallels:* 'as the Figure 'ABCD, whose sides AB, CD; 'and AC, BD, are parallels.

34. *The Diameter of a Parallelogram is a right line drawn from one angle to another: as BC.*

35. *The Complements are the two small Parallelograms, through which the Diameter does not pass: as AFEH, and GDIE.*

DEMANDS, *or* SUPPOSITIONS.

1. 'TIS suppos'd, that a right line may be drawn from any point whatsoever to another.

2. That a right line may be continu'd to what length you please.

3. That

3. That from a Center given a Circle may be defcrib'd at any diftance whatfoever.

MAXIMES, *or* AXIOMES.

1. THofe quantities that are equal to a third, are equal betwixt themfelves.

2. If equal quantities be added to thofe that are equal, the products will alfo be equal.

3. If equal quantities be taken away from thofe that are equal, the remainders will be equal.

4. If you add equal parts to quantities unequal, they will remain unequal.

5. If from equal quantities you take away unequal parts, the remainders will be unequal.

6. Quantities that are double, triple, quadruple, *&c.* in refpect of the fame, are equal among themfelves.

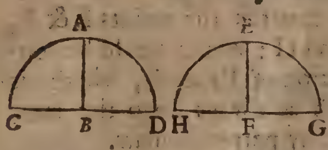
7. Thofe quantities are faid to be equal, which being apply'd one to the other, neither exceeds.

8. Equal lines and angles being plac'd one upon another, do not furpafs each other.

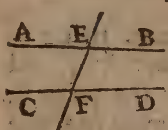
9. The whole is greater than its part.

10. All right angles are equal to one another.

'Let



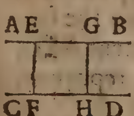
‘ Let the two
‘right angles pro-
‘ pos'd be ABC,
‘ EFH, I say they
‘ are equal. For if
‘ two equal circles CAD, HEG, be describ'd
‘ from the centers B and F; the fourth parts of
‘ those circles CA, HE, which are the measures
‘ of the angles, ABC, EFH, will be equal:
‘ therefore the angles ABC, EFH, having e-
‘ qual measures, will be equal.



The eleventh Maxime of *Euclid* is to this effect. If two lines AB, CD, being cut by a third EF, make the internal angles, BEF, DFE, less than two right angles; the lines AB, CD being produc'd, will at length concur towards the points B and D.

‘ Which, though it be true, is not clear
‘ enough to be reciv'd for a Maxime: therefore
‘ I have substituted another in its place.

11. If two lines be parallel, all the perpendiculars contain'd betwixt them will be equal.

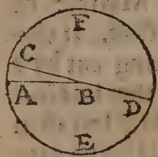


‘ As for example, if the lines AB,
‘ CD, are parallels, the perpendi-
‘ cular lines FE, HG, are equal.
‘ For if EF was greater than GH,
‘ the lines AB and CD, would be more remote

'from each other towards the points E & F, 'than towards G and H; which would be contrary to the definition of Parallels, where tis 'said, they are such as always keep the same 'distance, measur'd by perpendiculars.

12. Two right lines cannot enclose any space; that is to say, they cannot encompass it on all sides.

13. Two right lines cannot have one common segment.



'By which I mean, that two 'right lines, *suppose* A B, and 'CB, meeting at the point B, 'cannot together make one sole 'line BD; but cutting one another separate again immediately after their rencounter. For, if you describe a circle from the point B as a center, 'AFD will be a semicircle, because the right 'line ABD, passing through the center B, will 'divide the circle into two equal parts. The 'segment CED will be also a semicircle, because CBD will be also a right line, and will 'pass through the center B: therefore the segment CFD will be equal to the segment AED. 'the part to the whole; which is repugnant 'to the ninth Maxime.

Adver-

ADVERTISEMENT.

'THere are two ſorts of Propoſitions: In
'ſome we have nothing but the bare Spe-
'culation of a Truth, without deſcending to
'Practice; which we call *Theorems*; in the
'other ſomething is propos'd to be done, and
'thoſe are call'd *Problems.*

'The firſt number of the quotations denotes
'the Propoſitions, the ſecond the Book. As
'*by the* 2. *of the* 3. that is, by the ſecond Pro-
'poſition of the third Book: but if only one
'number occur, it ſignifies ſuch a Propopoſi-
'tion of the book you are then upon.

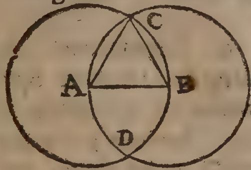
PROPOSITION I.

A PROBLEM.

To draw an Equilateral Triangle upon any line given.

LET the line AB be propos'd for the baſe of an Equilateral Triangle; from the center A at the diſtance AB deſcribe the circle BCD; and likewiſe from the center B at the diſtance BA deſcribe the circle DAC cut-

ting the former at the point C. Then draw the lines AC and BC; and all the ſides of the triangle ABC will be equal.



Demonſtration.

The Lines AB, and AC being drawn from the ſame center A to the circumference of the circle BCD are equal, *by the Definition of a Circle*; the lines BA, and BC are likewiſe equal being drawn from the center B to the circumference of the circle CAD. Laſtly the lines AC and BC being equal to the ſame line AB,[a] are alſo equal between themſelves. All the three ſides therefore of the triangle ABC are equal.

[a] max:1.

The USE.

'The deſign of *Euclid* in 'placing this Problem here 'was only to demonſtrate the 'two following Propoſitions. 'But it may be alſo further 'ſerviceable for the meaſuring 'an inacceſſible line, as for example, the line 'AB, which by reaſon of a River or Precipice 'cannot be approach'd. In ſuch a caſe make 'a ſmall Equilateral Triangle BDE, either of 'wood or copper, or the like; and having 'placed it Horizontally upon B, obſerve the

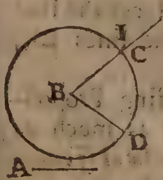
point

'point A, by the side BD, and any other point
'C, by the side BE. Then transfer your Tri-
'angle along the line BC, and place it upon
'divers parts of the same line; till at length
'you find a point C, upon which placing the
'Triangle you shall see the point B, by the side
'CG, and the point A by the side CF. I say
'the lines CB and CA are equal; so that by
'measuring the line BC, you may know the
'line AB. I might further demonstrate that
'the lines AB, and BC are equal; but let it
'suffice that in this Proposition, you are taught
'the way of making an Instrument proper to
'take the dimensions of an inaccessible line.

PROPOSITION II.

A PROBLEM.

From a point given to draw a line equal to another line given.



LET the point propos'd be B, from which a line is to be drawn equal to the line A. Take with the Compass the length of the line A, and at that interval, making B the Center, describe the circle CD. Drawing then from the

the point B to which ſide you pleaſe, a line BI, or BD, 'tis evident it will be equal to the line A.

'*Euclid* propoſes a more myſterious and intricate method of demonſtrating this Propoſition; but in practice we alwayes make uſe of this; in as much as, having taken with the compaſs the line A, 'tis as eaſy deſcribing a circle from the center B, as from the center A.

PROPOSITION III.

A Problem.

From a greater line to take a part equal to a leſs.

Suppoſe you were to take from the line BC, a part BI, equal to the line A. Take betwixt the points of the compaſs the length of the line A, and at that diſtance, from the center B deſcribe a circle, which ſhall cut the line BC at the point I. 'Tis certain the lines BI, and A, are equal.

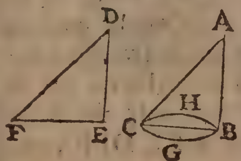
'The *Uſe* of theſe two preceding Propoſitions is ſufficiently evident; for as much as we are frequently oblig'd in *practical Geometry* to draw one line equal to another; and to take a part of a greater line equal to a line that is leſs.

PROP.

PROPOSITION IV.

A THEOREM.

If two Triangles have two sides equal, each to the other respectively, and the angles also, form'd by those two sides, equal; their bases and other angles will be equal.



LET the triangles ABC, DEF, have two sides equal each to the other respectively; that is to say, let AB be equal to DE, and AC to DF; and let the angles BAC, EDF, form'd by those sides be also equal: I say, the bases BC, EF, are equal, and the angles ABC, DEF; ACB, DFE, are equal; and lastly, the whole triangles equal in all respects.

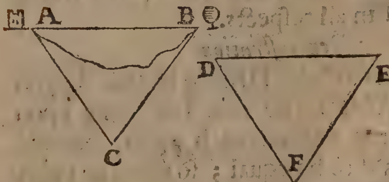
Demonstration.

Suppose the triangle DEF to be plac'd upon the triangle ABC: the side DE being upon AB, they will not exceed each other, because they are suppos'd to be equal; so that the point E will be upon B, and the point D upon the point A. For the same reason the line DF will fall upon AC. For if it should fall on the outside

ſide of it, the angle EDF would be greater than the angle BAC; and if it ſhould fall within AC, the angle EDF would be leſs: and yet they are ſuppos'd to be equal. Therefore ſince the point D is upon the point A, and the line DF falls upon the line AC, to which it is equal, they will not exceed each other, but the point F will fall upon C. Laſtly, ſince the points E and F of the line EF, fall upon B and C; the line EF will fall upon BC; becauſe it can neither fall higher as in BHC, nor lower as in BGC; for then two right lines would encloſe ſpace; which is contrary to the twelfth Maxime. Therefore the two triangles do not at all exceed each other; but not only the baſes BC, EF, but alſo the angles ABC, DEF; and ACB, DFE, are equal.

Coroll. An Equilateral triangle hath all its angles equal.

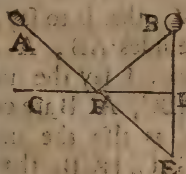
The USE.



'Suppoſe I 'was to meaſure an inacceſſible line AB. I obſerve 'from the point C the points A, and B; and 'then meaſure the angle C. This done, placing 'a board horizontally, and obſerving ſucceſſive- 'ly

'ly by a rule the points A and B, I draw two
'lines according to the rule, which make the
'angle C ; and meaſure with a yard the lines
'AC, and BC, which are ſuppos'd acceſſible.
'Then going into an open field, and placing
'my board again horizontally upon the point
'F, and obſerving the lines that I drew upon it,
'I make an angle DFE equal to the angle C.
'I make likewiſe FD, FE, equal to CA, CB.
'Then according to this propoſition the lines
'AB, and DE, are equal. So that meaſuring
'by the yard the acceſſible line DE, I ſhall
'know AB, which is inacceſſible.

Another USE.



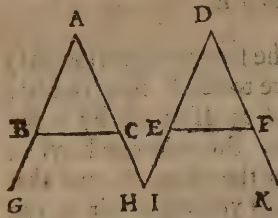
'The ſame propoſition may
'ſerve to teach how to hit a
'bowl at Billiards by refle-
'xion. Suppoſe one bowl to
'be at the point A, and that
'which you would hit at the
'point B, and CD the Billiard table. Imagine
'then a perpendicular BDE ; and take the
'line DE equal to BD. I ſay, if you direct
'the bowl from the point A to E, the reflexion
'will carry it to B. For in the triangles BFD,
'EFD, the ſide FD being common, and the
'ſides BD and DE equal ; the angles BFD,
'EFD are equal, by this propoſition. The an-

‘gles AFC, DFE, being oppoſite, are alſo equal, ‘as I ſhall demonſtrate hereafter. Therefore ‘the angle of Incidence AFC, is equal to the ‘angle of Reflexion BFD ; and by conſequence the Reflexion will be by AFB.

PROPOSITION V.

THEOREM.

In Iſoſceles, or Equicrural triangles, the angles that are above the Baſe are equal: as alſo thoſe that are below it.



Let the Iſoſceles be ABC, that is to ſay, let the ſides AB, AC be equal. I ſay the angles ABC, ACB are equal ; as alſo the angles GBC, HCB, that are below the baſe BC. Suppoſe another triangle DEF, having the angle D equal to the angle A ; and the ſides DE, DF, equal to AB, AC. Since the ſides AB, AC are equal, all the four lines AB, AC, DE, DF will be equal.

Demonſtration. Since the ſides AB, DE ; AC, DF, are equal ; as alſo the angles A, and D : if the triangle DEF be plac'd upon ABC, they will

will not exceed each other, but the line DE will fall upon AB; DF upon AG; and EF upon BC (by the 4*th*.) therefore the angle DEF, will be equal to ABC. And becauſe one part of the line DE falls upon AB, the whole line DI will be upon AG; otherwiſe two right lines would have a common ſegment; therefore the angle IEF will be equal to GBC. Suppoſe then the triangle DEF turn'd, and apply'd another way to the triangle ABC, that is to ſay, ſo as DF may fall upon AB, and DE upon AC. Since the four lines AB, DF; AC, DE, are equal; as alſo the angles A and D: the triangles will likewiſe agree this way, and the angles ACB, DEF; HCB, IEF, will be equal. Now by the firſt comparing them it appear'd, that the angle ABC was equal to the angle DEF; GBC to IEF: therefore the angles ABC, ACB being equal to the ſame DEF; and GBC, HCB, alſo equal to the ſame IEF, they are equal among themſelves.

max 13

max 1

'I was unwilling to make uſe of *Euclid*'s 'demonſtration, becauſe being very difficult, 'it might diſcourage beginners.

PROPOSITION VI.

Theorem.

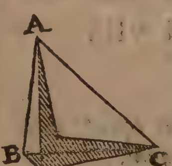
If two angles of a triangle be equal, the triangle will be an Isosceles.

LET the angles ABC, ACB of the triangle ABC be equal: (*see Fig. preced.*) I say it is an Isosceles; that is to say, the two sides AB, AC, which are opposite to the equal angles, are equal. Suppose the triangle DEF to have a base EF equal to BC, and the angle DEF equal to ABC, as also DFE equal to ACB: since the angles ABC, ACB are suppos'd to be equal, all the four angles ABC, ACB, DEF, DFE, will be equal. Suppose again therefore the base EF to be plac'd upon the base BC, so that the point E lie upon the point B, the bases being suppos'd equal it is evident they will not exceed each other. Further, the angle E being equal to the angle B, and the angle F to the angle C; the line ED will fall upon the line BA, and FD upon CA: so that the lines ED and FD will meet at the point A. From whence it follows, that the line ED is equal to BA.

Let then the triangle DEF be turn'd to the other side, and be applied another way to the tri-

triangle ABC: that is to ſay, ſo that the point E lie upon C, and F upon B: the baſes BC, FE will perfectly agree, being ſuppos'd to be equal: and becauſe the angles F, and B; E, and C, are alſo ſuppos'd to be equal, the ſide FD will fall upon BA, and ED upon CA; and the point D upon A. Therefore the lines AC, DE will be equal. Whence it follows, that the ſides AC, AB are equal between themſelves, being equal to the ſame ſide DE.

The USE.



'This Propoſition may ſerve 'for taking the dimenſions of 'any ſort of inacceſſible lines. ''Tis ſaid that *Thales* was the 'firſt that meaſur'd the heighth 'of Obelisks by their ſha-'dows: it may be done by this Propoſition. 'For if you were to meaſure the height of the 'Obelisk AB; do but expect till the Sun be 'elevated 45 degrees above the Horizon; that 'is to ſay, till the angle ACB be 45 degrees; 'and, *by the ſixth Propoſition*, the ſhadow BC 'will be equal to the Obelisk AB. For ſince 'the angle ABC is a right angle, and the angle 'ACB half a right one, or of 45 degrees; the 'angle CAB will be half a right one, as I ſhall 'prove hereafter. Therefore the angles BCA,

'BAC,

'BAC, are equal: and *(by the 6.)* the ſides AB, 'BC, are alſo equal. I can alſo meaſure the ſame 'height without making uſe of the ſhadow, by 'taking a ſtand ſo far from the point B, as that 'the angle ACB may be half a right angle, 'which may be known by a *Quadrant*.

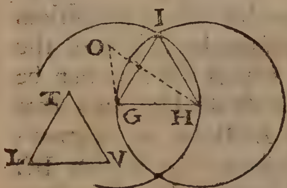
'These Propoſitions are of frequent uſe in 'Trigonometry, and in all other tracts,

'The *ſeventh* Propoſition may be omitted, 'becauſe tis of no other uſe but to demonſtrate 'the *eighth*, which may be done without it.

PROPOSITITION VIII.

Theorem.

If two Triangles have all their ſides equal, their oppoſite angles will alſo be equal.



LET the ſide GI be equal to LT; HI, to VT; GH, to LV; I ſay, that the angle GIH, will be equal to the angle LTV; IGH, to the angle L; and IHG, to the angle V. From the center H, at the diſtance HI, deſcribe the circle IG; and from the center G, at the diſtance GI, the circle HI.

Demon-

Demonstration.

Suppose the line LV brought upon HG; they would not exceed each other, because they are suppos'd to be equal. I add, that the point T will fall precisely upon the point I: For it ought to reach precisely to the circumference of the circle IG, because by the supposition the lines HI and VT are equal. It ought in like manner to reach to the circumference of the circle IH, because the lines GI and LT are equal. So then it will light upon the point I, being the point where those two circles cut each other. Indeed if it fell any where else, as upon O, the line HO, that is to say VT, would be greater than HI; and the line GO, that is LT, would be less than GI; which is against the supposition. Whence I conclude, that the triangles will exactly correspond, and the angle GIH be equal to the angle LTV.

The USE.

'This Proposition is necessary for the proof 'of those that follow. And further, when we 'cannot take the measure of an angle, because, 'the lines meeting in a solid body, we cannot 'apply our Instruments to it; we must take 'the three sides of the triangle, and make an-'other upon a paper, whose angles we may 'measure. This is a very ordinary practice in

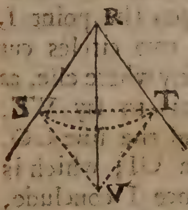
Gro-

'*Gnomonicks*, or Dialling; and in the treatises
'concerning cutting pretious stones, so as to fit
'the pannels, and to retain the waters.

PROPOSITION IX.

PROBLEM.

To divide an Angle into two equal parts.



LET the angle SRT be propos'd to be divided into two equal parts. Take the Compass, and from the center R, at any distance, draw the arch ST; cutting off two equal lines RS, RT. Then draw the right line ST; and (*by the* 1.) describe an equilateral triangle STV. I say, the line VR divides the angle into two equal parts: that is to say, the angles VRT, and VRS, are equal.

Demonstration.

The triangles VRS, and VRT, have the side VR common; and the side RT was taken equal to the side RS: the base also SV, is equal to VT, because the triangle SVT is equilateral. Wherefore (*by the* 8.) the angles SRV, VRT, are equal.

The

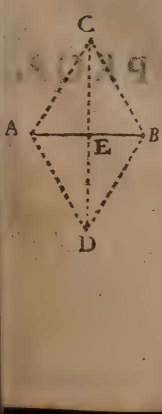
The USE.

'This propoſition is very uſeful to divide 'the fourth part of a circle into degrees : for tis 'the ſame thing to divide an arch, as an angle 'into two equal parts ; and the line RV does 'both, that is, it divides both the arch ST, and 'the angle SRT. Having therefore apply'd the 'ſemidiameter to the fourth part of a circle, 'you cut off an arch of 60 degrees, which divi- 'ded equally gives an arch of 30 ; and that 'again divided, makes one of 15 degrees. Tis 'true, to finiſh this diviſion, we muſt divide an 'arch into three equal parts, but that is not to 'be done Geometrically. Pilots alſo divide 'the Compaſs into 32 winds by the help of this 'Propoſition only.

PROPOSITION X.

A PROBLEM.

To divide a right line into two equal parts.



Suppoſe the line AB was to be divided into two equal parts ; upon the line AB deſcribe an equilateral triangle AB C, (*by the* 1.) and divide the angle ACB into two equal parts by the line DC, (*by the* 9.) I ſay the

the line AB is divided equally at the point E; that is to ſay, the lines AE and EB are equal.

Demonſtration.

The triangles ACE, and BCE have the ſide CE common, and the ſides CA and CB are equal, becauſe the triangle ACB is equilateral: and the angle ACB being divided equally, the angles ACE and BCE are alſo equal. Therefore (*by the* 4.) the baſes AE and BE are equal.

The USE.

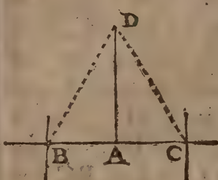
'Great uſe is made of this Propoſition, ordinary practices frequently requiring us to divide a line in the middle; which *Geometricians* require ſhould be done exactly at the firſt daſh, by a method that is infallible, and not by eſſays. This practice is likewiſe principally uſeful for dividing meaſures into leſs parts.

PROP.

PROPOSITION XI.

A PROBLEM.

To draw a Perpendicular to a line given, upon a point of the same line.



Suppose you were to raise a perpendicular upon the point A of the line BC. Take two equal lines AB by ye 10 and AC on both sides the point A, and make an equilateral triangle BDC upon the line BC, (*by the* 1.) I say the line AD is perpendicular, that is to say, the angles BAD and CAD are equal.

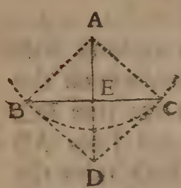
Demonstration.

The triangles BAD, and CAD have the side AD common, the sides AC and AB are equal, and the bases BD and DC also equal: therefore (*by the* 8.) the angles BAD, and CAD, are equal; and (*by the* 10. *def.*) the line AD perpendicular to BC.

PROPOSITION XII.

A PROBLEM.

To draw a perpendicular to a line given, from a point, which is out of the line.



IF you would draw a perpendicular to the line BC, from the point A: having ſet the foot of the compaſs upon A, deſcribe the circle BC, which ſhall cut the line BC, at the points B and C. *Then divide the line BC into two equal parts at the point E. I ſay the line AE is perdendicular to BC. Draw the lines AB, AC.

*by yᵉ 10

Demonſtration. The triangles BEA, and CEA, have the ſide AE common; and the ſides EC and EB equal, the line BC having been equally divided at the point E; *the baſes AB and AC, being drawn from the center A to the circumference BC, are likewiſe equal: therefore the angles AEB, and AEC, are equal, (*by the* 8.) and the line AE perpendicular, (*by defin.* 10.)

*by yᵉ 15 defin:

The method, in practice, of dividing the line BC in the middle, is to deſcribe two arches at D, at the ſame interval, from the centers B and C.

The

The USE.

'We have need of a Plummet or Squaring-
'line almost in all our operations: no angles
'are of use in buildings but the right; and all
'chairs, benches, tables, buffets, and other move-
'ables, are fram'd by the square. No survey of
'Land can be taken without making use of per-
'pendicular lines; nor can *Dialling* be per-
'form'd without them. The Carpenter's Le-
'vel contains a right angle, and the same is
'preferr'd before any other, especially by the
'*French*, in Fortifications. Lastly, not only
'*Mathematicians*, but also the greatest part
'of practical Artisans, require that we should
'know how to draw a perpendicular.

PROPOSITION XIII.

A THEOREM.

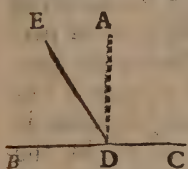
One line falling upon another makes with it either two right angles, or two angles equal to two right ones.

LEt the line AD fall upon BG; I say, 'twill make with it either two right angles; or two angles, one obtuse, and the other acute, which joyn'd together shall be of equal value with two right ones.

Demon-

Demonstration.

Suppose the line AD to fall perpendicularly upon BG, then tis evident (*by defin.* 10.) that the angles ADB, and ADG, are equal, and by consequence right angles. Or,



Secondly, suppose the line ED not to fall perpendicularly upon BC, and draw a perpendicular AD (*by the* 11.) the angles ADB, and ADC are right angles, which are of equal value with the three angles ADC, ADE, EDB. But the obtuse angle EDC, and the acute angle EDB, are of equal value with the three angles ADC, ADE, and EDB: therefore the angles EDC, and EDB, are of equal value with two right ones.

This Proposition may be more easily demonstrated by describing a semicircle from the center D vpon the line BC. For the angles EDB, and EDC, will require a semicircle for their measure, which is the measure of two right angles, as I have shown before; *in the* 8. *definition.*

Corollary 1. If the line AD falling upon BC, make one right angle ADC; it is evident the other, ADB, will be also a right angle.

Coroll. 2. If the line ED, falling upon BC, make the angle EDB acute; the angle EDC will be obtuse.

The

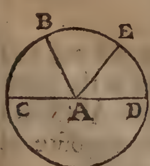
The USE.

'By this means, when we know one of the 'angles which is made by one line falling upon 'another, we know alſo the other: as for ex-'ample, if the angle EDB be one of 70 de-'grees, taking away ſeventy from 180, there 'will remain 110 for the angle EDC. This 'operation does frequently occur in *Trigono-'metry*; and alſo in *Aſtronomy*, for finding the 'eccentricity of the circle through which the 'Sun annually paſſes.

PROPOSITION XIV.

A THEOREM.

If two lines meeting together at the ſame point of another line, make with it two angles equal to two right ones; they will make but one and the ſame line.



SUppoſe the lines CA, and DA, to meet at the point A of the line AB; and that the angles adjoyning, CAB, and BAD, are equal to two right ones. I ſay, the lines CA and DA are but one and the ſame line; ſo that CA being continued, will fall preciſely upon AD.

Imagine

Imagine, if you pleaſe, that CA continu'd will paſs on to E, and from the center A deſcribe a circle.

Demonſtration.

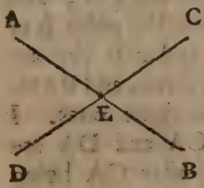
If you ſay that CAE is a right line, the arch CBE will be a ſemicircle. But tis ſuppos'd, that the angles CAB, and BAD are equal to two right ones, and that therefore their meaſure is a ſemicircle. Therefore the arches CBE, and CBD will be equal; which is impoſſible, one being a part of the other. Therefore the line CA being continu'd, will make but one and the ſame line with AD.

PROPOSITION XV.

A THEOREM.

*If two right lines cut each other, the oppoſite angles * at the top will be equal.*

* κατὰ κορυφήν, *Eucl.* au ſommet. *Gall.*



LET the lines AB and CD cut each other at the point E: I ſay, the angles AEC, and DEB, which are oppoſite at the top, are equal.

Demonſtration,

The line CE falling upon the line AB, makes the

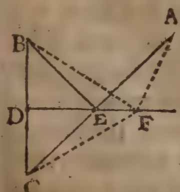
the angles AEC and CEB equal to two right ones, (*by the* 13.) In like manner the line BE falling upon the line CD, makes the angle CEB and BED equal to two right ones. Therefore the angles AEC, CEB, taken together, are e- qual to the angles CEB, BED; therefore taking away the angle CEB from both, the angle AEC will remain equal to DEB, (*by the* 3. *Maxime.*) max

Coroll. 1. If two lines DE, and EC, concurring at the same point E of the line AB, form with it the opposite angles AEC, DEB equal, DE and EC make but one right line.

Demonstration.

The line EC falling upon the line AB, makes the angles AEC, and CEB equal to two right ones, (*by the* 13.) Tis suppos'd likewise that the angle DEB is equal to the angle AEC. Therefore the angles DEB, BEC, are equal to two right ones. And (*by the* 14.) the lines CE and ED make but one right line.

The USE.



'The two preceding Propositions are made use of to 'prove, that two lines make 'but one total. As for example, in Catoptricks or Perspectives, where that is required 'to prove, that of all the lines

 that

'that can be drawn by reflexion from the point
'A to the point B, thoſe are the ſhorteſt,
'which make the angle of Incidence equal to
'the angle of Reflexion. As for example; if the
'angles BED and AEF be equal, the lines AE,
'and EB, are ſhorter than AF, and FB. †From
'the point B draw a perpendicular BD, [2] and
'make the lines BD and CD equal; then draw
'EC, and FC. Firſt in the triangles BED and
'CED the ſide DE is common; and the ſides
'BD, and DC being equal, as alſo the angles
'BDE, and CDE; the baſes BE, and CE will
'be equal; as alſo the angles BED, and DEC,
'(*by the* 4.) In like manner I may prove, that
'BF, and CF are equal.

† p 12
2 10

Demonſtration.

'The angles BED and DEC are equal, and the
'angles BED and AEF are ſuppos'd likewiſe to
'be equal; therefore the oppoſite angles DEC
'and AEF will be equal; and (*by the Coroll. of*
'*the* 15.) AEC one right line; and by con-
'ſequence AFC is a triangle, of which the ſides
'AF and FC muſt be longer than AEC, that is
'to ſay, than AE, and EB. But the lines AF
'and FC are equal to the lines AF, and FB;
'therefore the lines AF and FB are longer than
'the lines AE and EB. And ſince natural cau-
'ſes always act by the ſhorteſt lines, the Refle-
'xion will always happen in ſuch a manner, that
'the angles of Reflexion and Incidence may be
'equal.

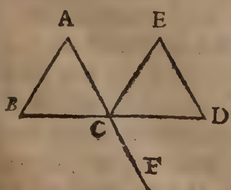
Fur-

'Further, becaufe we can eafily prove, that
'all the angles that can be made upon a plane
'about the fame point, are equal to four right
'angles; for as much as in the firft figure of
'this propofition, the angles AEC and AED
'are equal to two right ones, as alfo BEC and
'BED to two more: we make a general rule
'to determine what Polygones may be joyn'd in
'paving a Hall. Accordingly we fay, that four
'fquares, fix triangles, and three hexagones,
'may be ufed for that purpofe; and that there-
'fore Bees are always obferv'd to make their
'little cells of the laft, that is, of figures con-
'fifting of fix fides.

PROPOSITION XVI.

A THEOREM.

The external angle of a triangle is greater than either of the internal oppofite angles.



PRoduce the fide BC of the triangle ABC: I fay the external angle ACD, is greater than either of the internal oppofite angles, ABC, or BAC. Suppofe the triangle ABC to be mov'd along the line BD,

and carry'd into the place of CED.

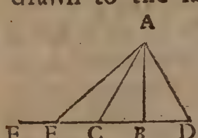
Demonstration.

Tis impossible that the triangle ABC should be so mov'd, but the point A must come into the place of the point E; and then twill appear, that the angle ECD, that is to say, ABC, is less than the angle ACD: therefore the internal angle ABC is less than the external ACD.

Tis likewise easie to prove, that the angle A is less than the external angle ACD: for having prolong'd the side AC as far as F, the opposite angles BCF, and ACD, are equal (*by the* 15.) Therefore causing the triangle ABC to slide along the line ACF, I shall demonstrate, the angle BCF to be greater than the angle A.

The USE.

We may draw from this proposition many 'most useful conclusions. As first, that from a 'point given only one perpendicualar can be 'drawn to the same line. For example, Sup'pose the line AB to be 'perpendicular to the line 'BC: I say, that AC will 'not be perpendicular; be'cause the right angle



'ABD must be greater than the internal angle 'ACB; therefore ACB cannot be a right angle, 'nor AC a perpendicular.

Se-

'Secondly, that from the same point A can-
'not be drawn more than two equal lines; for
'example, AC, and AD; and if you draw a
'third as AE, it will not be equal to the for-
'mer. For since AC and AD are equal, the an-
'gles ACD, and ADC, are equal, (*by the* 5.)
'but in the triangle AEC, the external angle
'ACB is greater than the internal AEC: and
'therefore likewise the angle ADE, is greater
'than the angle AED. Therefore the lines AE,
'and AD, are not equal; nor by consequence
'AC and AE.

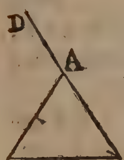
'Thirdly, that if the line AC makes the an-
'gle ACB acute, and ACF obtuse, the perpen-
'dicular drawn from the point A will fall on
'the side of the acute angle. For if you say
'that AE is a perpendicular, and that AEF is a
'right angle; the right angle AEF would be
'greater than the obtuse ACE. These conclu-
'sions are serviceable for measuring Parallelo-
'grams, Triangles, and Trapesia, and to re-
'duce them into rectangular figures.

PROP.

PROPOSITION XVII.

A THEOREM.

Any two angles of a triangle are less than two right ones.


B C

LET the triangle be ABC; I say, that any two of its angles taken together, as BAC, and BCA, are less than two right ones. Produce the side CA to the point D.

Demonstration.

The internal angle C, is less than the external BAD, (*by the* 16.) Add therefore to both the angle BAC; the angles BAC, and BCA, will be less than the angles BAC, and BAD; yet those are but equal to two right ones, (*by the* 13.) therefore the angles BAC, and BCA, are less than two right ones.

After the same manner I can demonstrate the angles ABC, and ACB, to be less than two right ones, by producing the side BC.

Coroll. If one angle of a triangle be a right, or obtuse, angle, the others are acute.

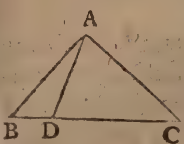
" This Proposition is necessary to demon-
" strate those that follow.

PROP.

PROPOSITION XVIII.

A Theorem.

In every triangle whatſoever the greateſt ſide is oppos'd to the greateſt angle.



Suppoſe the ſide BC of the triangle ABC, to be greater than the ſide AC: I ſay, the angle BAC, that is oppos'd to the ſide BC, is greater than the angle B, which is oppos'd to the ſide AC. Cut the line BC in D, ſo that CD may be equal to AC; then draw the line AD.

Demonſtration.

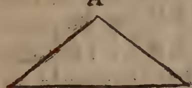
Since the ſides AC, and CD, are equal, the triangle ACD will be an *Iſoſceles*, and (*by the* 5.) the angles CDA, and CAD, equal. Now the whole angle BAC is greater than the angle CAD: therefore the angle BAC is greater than the angle CDA; which yet, being an external angle in reſpect of the triangle ABD, is greater than the internal B, (*by the* 16.) Therefore the angle BAC is greater than the angle B.

PROP.

PROPOSITION XIX.

A THEOREM.

In every triangle the greateſt angle is oppos'd to the greateſt ſide.



LET the angle A of the triangle BAC, be greater than the angle ABC. I ſay, the ſide BC, which is oppos'd to the angle A, is greater than the ſide AC, that is oppos'd to the angle B.

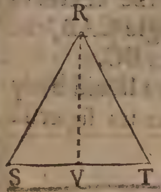
Demonſtration.

If the ſide BC be not greater than the ſide AC, tis either equal; and then the angles A and B would be equal, (*by the* 5.) which is contrary to the ſuppoſition: or leſs; and if ſo, the ſide by ye 18 AC being greater than BC, the angle B would be greater than the angle A, though the contrary be ſuppos'd. It remains therefore that the ſide BC be greater than the ſide AC.

The USE.

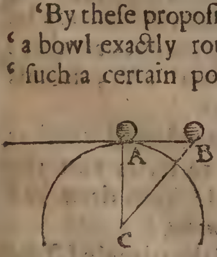
'We may prove from theſe propoſitions, 'not only that no more than one perpendicular can

‘can be drawn from the same point to the same ‘line; but also that it is the shortest of all. As



‘for example; if the line RV ‘be perpendicular to ST, it will ‘be less than RS: because the ‘angle RVS being a right angle, ‘the angle RSV will be an acute, ‘(*by the Coroll. of the* 17.) and ‘the line RV will be less than ‘RS, (*by the preceding.*) Therefore *Geometri-* ‘*cians* do always make use of a perpendicular, ‘when they take the dimensions of any thing, ‘and reduce irregular figures to such as have ‘one or more right angles. I add, that it being ‘impossible that more than three perpendiculars ‘should meet at the same point, it cannot be ‘imagin'd that there should be more than three ‘species or kinds of quantity, a line, a superfi- ‘cies, and a solid body.

‘By these propositions we likewise prove, that ‘a bowl exactly round cannot rest, but upon ‘such a certain point. For example; let the ‘line AB represent a plane,



‘and C the center of the ‘earth, and that CA be ‘drawn perpendicular to ‘the line AB; I say, that a ‘bowl being plac'd upon ‘the point B, cannot rest there. For a heavy ‘body cannot rest, when it may descend. Now

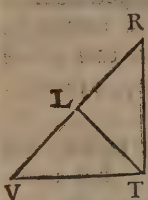
'the bowl B moving towards A continually de-
'scends, and approaches nearer the center of
'the earth C; becaufe in the triangle CAB, the
'perpendicular CA is fhorter than BC.

'In like manner we prove, that a liquid body
'muft flow from B to A, and that its fuperfi-
'cies muft be round.

PROPOSITION XX.

A Theorem.

Any two fides of a triangle taken together are greater than the third.



I fay that the two fides TL, LV, are greater than the fide TV. Some men prove this Propofition by the definition of a right line, which is the fhorteft that can be drawn from one point to another: therefore the line TV, is lefs than the two lines TL and LV.

But it may alfo be demonftrated another way. Continue the fide VL to R, fo that the lines LR, and LT be equal; then draw the line RT.

Demonftration.

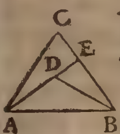
The fides LT, and LR, of the triangle LTR, are equal; therefore the angles R, and LTR, are equal, (*by the* 5.) But the angle RTV is greater

greater than the angle RTL: therefore the angle RTV is greater than the angle R: and (*by the* 19.) in the triangle RTV, the ſide RV, that is to ſay, the ſides LT and LV, are greater than the ſide TV.

PROPOSITION XXI.

A THEOREM.

If a ſmall triangle be deſcrib'd within a greater, upon the ſame baſe, the ſides of the ſmall one will be leſs than thoſe of the greater; but they will form a greater angle.



LET the ſmall triangle ADB be deſcrib'd within the triangle ACB, upon the ſame baſe AB. I ſay firſt, the ſides AC and BC are greater than the ſides AD and BD. Continue the line AD to E.

Demonſtration.

In the triangle ACE, the ſides AC and CE, are greater than the ſide AE alone, (*by the* 20.) Therefore adding to them the ſide EB; the ſides AC, and CEB, are greater than the ſides AE, and EB, In like manner in the triangle DBE, the two ſides BE and ED are greater than the ſide BD alone, and adding the ſide AD,

the ſides ADE, and EB, will be greater than AD and BD.

I ſay further, that the angle ADB is greater than the angle ACB: for the angle ADB is an external angle in reſpect of the triangle DBE, and therefore greater than the internal DEB (*by the* 16.) In like manner the angle DEB, being an external angle in reſpect of the triangle ACE, is greater than the angle ACE, therefore the angle ADB is greater than the angle ACB.

The USE.

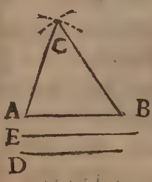
'By the help of this propoſition we demonſtrate in *Opticks*, that the Baſe AB view'd 'from the point C, will appear leſs, than when 'it is beheld from the point D; according to 'that principle, That quantities view'd under 'a greater angle will appear greater. Therefore 'tis, that *Vitruvius* adviſes, not much to leſſen 'the tops of very high Pillars, becauſe they, 'being ſo remote from our ſight, quickly 'appear ſlender enough without being diminiſh'd.

PROP.

PROPOSITION XXII.

A THEOREM.

To describe a triangle, whose sides shall be equal to three sides given, provided that any two of them be greater than the third.



LET it be propos'd to describe a triangle, whose sides shall be equal to three lines given, AB,D,and E. Measure with the compass the line D, and setting one foot thereof upon the point B make an arch, Then take the line E, and placing the foot of your compass upon the point A make another arch, cutting the former at the point C. Which done draw the lines AC.and BC. I say that the triangle ABC, is such a one as you desire.

Demonstration.

The side AC is equal to the line E, because it reaches to the arch, which is drawn from the center A at the distance of the line E; and for the same reason the side BC is equal to the line D: therefore the three sides AC, BC, and AB, are equal to the lines E,D, and AB.

I added a Proviso, that the two lines should be greater than the third: because otherwise if the

the lines D and E were lefs than the line AB, the arches could not cut each other.

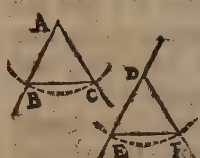
The USE.

'This Propofition may be ufeful for defcrib-'ing a figure equal or like to another: for hav-'ing divided that, which is propos'd to be 'equall'd or imitated, into triangles; and made 'other triangles, having equal fides with the 'former; we fhall have a figure exactly equal. 'But if we defire only one that is like, but lefs; 'as when we would defcribe a plain, or coun-'try upon paper: having divided it into trian-'gles, and meafur'd all their fides, we muft 'make fimilar triangles; giving to each of their 'fides fo many parts of a *Scale*, or line divided 'into equal parts, as the fides of the triangles 'propos'd have of yards or feet.

PROPOSITION XXIII.

A PROBLEM.

To make an angle equal to another at a point of a line given.



SUppofe you were to make an angle at the point A of the line AB, equal to the angle EDF. Defcribe from the points A and D as centers two

two arches BC, and EF, at the ſame wideneſs of the compaſs ; then take the diſtance EF, and having meaſur'd as much at BC, draw the line AC. I ſay the angles BAC, and EDF, are equal.

Demonſtration.

The triangles ABC, and DEF, have the ſides AB, and AC, equal to the ſides DE, and DF ; ſince the arches BC and EE were deſcrib'd with the ſame wideneſs of the compaſs: the baſes alſo BC and EF are equal ; therefore the angles BAC and EDF are equal, (*by the* 8.)

The USE.

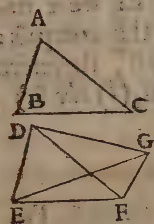
This Problem is ſo neceſſary in *Geodeſia*, *Fortifications*, *Perſpective*, *Dialling*, and all other parts of the Mathematicks, that the greateſt part of their Operations would be impoſſible, if we did not know how to make one angle equal to another, or of ſuch a number of degrees as we pleaſe.

PROP.

PROPOSITION XXIV.

A THEOREM.

Of two triangles, having each two sides equal to two of the other, that which has the greatest angle, has also the greatest base.



LET the triangles ABC, DEF, have the sides AB and DE; AC and DF equal; and let the angle BAC be greater than the angle EDF. I say, the base BC is greater than the base EF.

Make the angle EDG equal to the angle BAC, (*by the* 23.) x and

x prop 2

the line DG equal to AC; then draw the line EG. First the triangles ABC and EDG, having the sides AB and DE, AC and DG, equal, and the angle EDG equal to the angle BAC; their bases BC and EG will be equal (*by the* 4.) and the lines DG and DF being both equal to AC, will be equal betwixt themselves.

Demonstration.

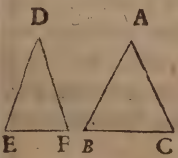
In the triangle DGF, the sides DG and DF being equal, the angles DGF and DFG will be equal, (*by the* 5.) But the angle EGF is less than the angle DGF, and the angle EFG is greater than the angle DFG. Therefore in the

the triangle EFG, the angle EFG will be greater than the angle EGF: and therefore (*by the* 18.) the line EG oppos'd to the greater angle EFG, will be greater than EF. Therefore BC, being equal to EG, is greater than EF.

PROPOSITION XXV.

A THEOREM.

Of two triangles, having each two sides equal to two of the other, that which has the greatest base, has likewise the greatest angle.



LET the two triangles ABC, DEF, have the sides AB, DE; and AC, DF, equal; and let the base BC be greater than the base EF. I say, that the angle A will be greater than the angle D.

Demonstration.

If the angle A be not greater than the angle D; it will be either equal, and then the bases BC, EF, will be equal, (*by the* 4.) or it will be less, and the base EF greater than the base BC, (*by the* 24.) but both are contrary to the supposition.

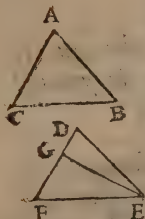
" These Propositions are necessary to de-
" monstrate those that come after.

H PROP

PROPOSITION XXVI.

A Theorem.

If one triangle has one side, and two angles, equal to to those of another triangle; tis equal to it in all respects.



LET the angles ABC, DEF; ACB, DFE, of the triangles ABC, DEF, be equal; and the sides BC, and EF, which are between those angles, also equal. I say, that the other sides are equal; for example, AC, and DF. Imagine, if you please, the side DF to be greater than AC; and cutting GF equal to AC draw the line GE.

Demonstration.

The triangles ABC, GEF, have the sides EF, BC; AC, GE, equal; the angle C is also suppos'd to be equal to F. Therefore (*by the* 4.) the triangles ABC, GEF, are equal in all respects; and the angles GEF, and ABC, are equal. But we suppos'd the angles ABC, DEF, to be equal: and so, the angles DEF, GEF, would be equal, that is, the whole to the part; which is impossible. Therefore the side DE will not be greater than the side AC, nor AC greater than DF,

DF, because the same demonstration may be made in the triangle ABC.

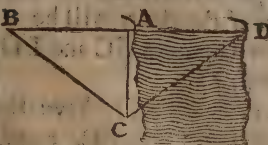
Again, suppose the angles A and D; C and F to be equal; and also the sides BC, and EF, oppos'd to the angles A and D, to be equal. I say, the other sides are equal. For if DF be greater than AC, cut GF equal to AC, and draw the line GE.

Demonstration.

The triangles ABC, GEF, having the sides EF, BC; FG, CA, equal, will (*by the* 4.) be equal in all respects; and the angles EGF, BAC, will be equal. But we suppos'd, that the angles A and D were equal, therefore the angles D, and EGF, must be equal, which is impossible, since the angle EGF, being the external angle in respect of the triangle EGD, must be greater than the internal D, (*by the* 16,) therefore the side DF is not greater than AC.

The USE.

'*Thales* made use of this proposition to measure inaccessible distances. For example: the



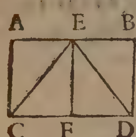
'distance AD being 'propos'd, he would 'draw from the point 'A, the line AC per- 'pendicular to AD: 'then describing a semicircle at the point C, 'would measure the angle ACD, and take an-

other

‘other equal to it on the other side, prolonging the line CB till it met with the line DA at the point B; and then demonstrated the lines AD and AB to be equal; so that measuring the line AB, which was accessible, he could know the other which was not. For the two triangles ADC, and ABC, have the right angles CAD, and CAB equal, the angles ACD, and ACB are also taken equal; and the side AC is common to both: therefore (*by the 26.*) the sides AD and AB are equal.

A LEMMA.

A line which is perpendicular to one of two parallels, is also perpendicular to the other.



‘Let the parallel lines be AB, and CD, and let EF be perpendicular to CD. I say, tis also perpendicular to AB. Cut the line CF equal to FD, and upon the points C and D raise two perpendiculars to CD, which, *by the definition of Parallels*, will be equal to FE; then draw the lines EC and ED.

Demonstration.

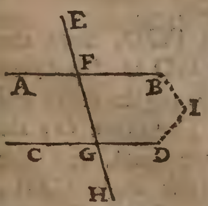
‘The triangles CEF, and FED, have the side FE common, the sides CF, FD, equal, * 10 def. and the angles CFE, and EFD,* right, and by consequence equal; therefore (*by the 4.*) the bases

'bases EC, ED, and the angles FED, FEC, and
'FCE, FDE, will be equal; the two last of
'which being taken away from the right angles
'ACF and BDF, leave the two angles ACE, & max
'and BDE, equal; therefore the triangles 3
'CAE, DBE, will have (*by the* 4.) the angles
'DEB, CEA, equal; which being added to the
'equal angles CEF, FED, make the angles
'FEB, and FEA, equal; therefore the line EF
'is perpendicular to AB.

PROPOSITION XXVII.

A Theorem

If a line, falling upon two others, makes with them the alternate angles equal, those two lines are parallel.



LET the line EH, falling upon the lines AB and CD, make with them the alternate angles AFG, FGD equal, I say first, the lines AB, and CD, will never concur, though continu'd as far as you please. For suppose them to concur in I, and that FBI, and GDI, are two right lines.

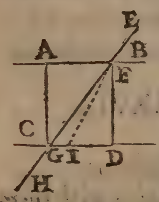
Demonstration.

If FBI, and GDI, be two right lines, FIG is

is a triangle; and (*by the* 16.) the external angle AFG, is greater than the internal FGI. They cannot therefore be equal, if the lines AB and CD ever concur.

'But because we have examples of some 'crooked lines, which never concur; and yet 'are not parallels, approaching still nearer and 'nearer to each other,

I say secondly, that if the line EH, falling upon the lines AB, and CD, makes the alternate angles AFG, and FGD equal; the lines AB, CD, are parallel, or in all respects equally remote from each other, so that the perpendiculars between them will be equal. From the point G draw the perpendicular GA to the line AB; and taking GD equal to AF, draw FD.



Demonstration.

The triangles AGF, and FGD, have the side *F*G common; the side GD is also taken equal to the side *AF*, and the angles A*F*G and *F*GD suppos'd to be equal. Therefore (*by the* 4.) the bases AG and *F*D are equal, and the angle CD*F* is equal to the right angle CAB; therefore *F*D is perpendicular. I add, that the line AB is parallel to CD: for the only parallel line that can be drawn from the point *F* to the line CD, ought to pass by the point A, according

cording to the definition of Parallels; which requires, that the perpendicular lines AG and *F*D be equal.

PROPOSITION XXVIII.

A THEOREM.

If a line, falling upon two others, makes the external angle, equal to the internal opposite angle on the same side; or the two internal angles on the same side equal to two right ones; those two lines will be parallel.

IN the precedent figure, suppose the line EH, falling upon AB, and CD, to make first the external angle E*F*B equal to the internal opposite angle on the same side FGD. I say, that the lines AB, and CD, are parallel.

Demonstration.

The angle E*F*B is equal to the angle A*F*G, being oppos'd to it at the top (*by the* 15.) and tis suppos'd that the angle *F*GD is also equal to the angle E*F*B; therefore the alternate angles A*F*G, *F*GD, will be equal; and (*by the* 27.) the lines AB, and CD, will be parallel.

I say in the second place, that if the angles B*F*G, and *F*GD, which are the internal angles on the same side, be equal to two right ones, the lines

lines AB and CD will be parallel.

Demonstr. The angles AFG and BFG are equal to two right angles, (*by the* 13.) and tis suppos'd that the angles BFG, and FGD, are also equal to two right angles; therefore the angles AFG, BFG, are equal to the angles BFG, and FGD; therefore taking away the angle BFG, which is common to both, the alternate angles AFG and FGD will be equal; and (*by the* 27.) the lines AB and CD will be parallel.

PROPOSITION XXIX.

A Theorem.

If a line cut two parallels, the alternate angles will be equal; the external angle will be equal to the internal opposite angle; and the two internals on the same side will be equal to two right angles..

LET the line EH [*see fig, preced.*] cut the two parallels AB, and CD; I say first, the alternate angles AFG, and FGD, are equal. From the points F and G draw the perpendiculars GA, and FD, which by the definition of Parallels are equal.

Demonstration.

In the rectangle triangles AFG, and FGD, the

the sides *FD* and AG being equal, as also the right angles A and D, and the side *FG* common to both: I say first, that the side GD is equal to *AF*. For if GD be greater; having cut the line DI equal to *AF*, and drawn the line *FI*; the triangles *AFG*, and *FDI*, would have their bases *GF* and *FI* equal,+ which is impossible. + 4
For since the angle D is a right angle, the angle *FID* is an acute, and *FIG* an obtuse, (*by the* 13.) therefore (*by the* 18.) in the triangle *FIG*, the side *FG* oppos'd to the obtuse angle, is greater than *FI*. Therefore DG is equal to *AF*; and the triangles *AFG* and *FGD*, having all their sides equal, will have the alternate angles *AFG* and *FGD* equal, as being oppos'd to the equal sides AG, and *FD*.

I say again, that the external angle *EFB* is equal to the internal EGD, because (*by the* 15.) it is equal to its opposite *AFG*, which is equal to its alternate *FGD*.

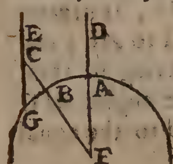
Lastly, since the angles *AFG* and *GFB* are equal to two right ones; taking away *AFG*, and substituting in its place its alternate *FGD*, the two internal angles *GFB*, and *FGD*, will be equal to two right angles.

The USE.

'*Eratostenes* found out by these Propositions
'a way of measuring the circuit or circumfe-

I rence

'rence of the Earth. In order to which he ſup-'pos'd two rays, proceeding from the center of 'the Sun to two points of the earth, to be phy-'ſically parallel; and alſo that at *Syene*, a town 'in the higher parts of *Egypt*, the Sun comes 'exactly to the Zenith upon the day of the Sol-'ſtice, obſerving the Wells there to be then 'illuminated to the very bottom: and likewiſe 'computed the diſtance between *Alexandria* 'and *Syene* by miles, or furlongs.



'Let us therefore ſuppoſe *Sy-'ene* to be at the point A, and '*Alexandria* at B, where we 'erect a ſtyle BC perpendicu-'lar to the Horizon; and let 'the two lines *DF* and EG re-'preſent the two rays proceeding from the cen-'ter of the Sun upon the day of the Solſtice, 'which are parallel to each other. DA, 'which paſſes by *Syene*, is perpendicular, that 'is, it paſſes through the center of the earth. 'Having obſerv'd by the perpendicular ſtyle 'BC the angle GCB, made by the ray of the 'Sun EG; I ſay, the rays DA and EG being paral-'lel, the alternate angles GCB and BFA are 'equal; by which means we have got the angle 'AFB, and its meaſure AB; which gives us in 'degrees the diſtance between *Alexandria* and '*Syene*. And having ſuppos'd it to be known 'in miles, the circumference of the earth may

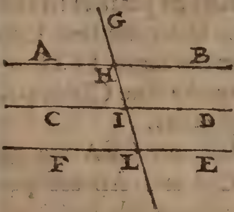
'be

'be found by the simple Rule of Three, saying, '*If so many degrees give so many miles, how many* '*will 360 give?*

PROPOSITION XXX.

A THEOREM.

Lines parallel to a third, are also parallel among themselves.



SUppose the lines AB, and *FE* to be parallel to the line CD: I say, they are parallel betwixt themselves. Let the line GL cut them all three.

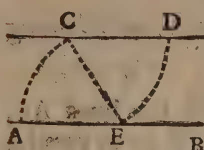
Demonstration.

For as much as the lines AB and CD are parallel, the alternate angles AHI, and HID, are equal (*by the 29,*) and because the lines CD and *FE* are also parallel, the external angle HID will be equal to the internal ILE; [*by the same*] therefore the alternate angles AHI, and ILE, will be equal, and the lines AB and *FE* parallel [*by the 27.*]

PROPOSITION XXXI.

A PROBLEM.

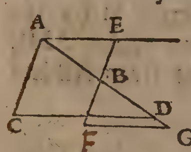
To draw a line parallel to another by a point given.



LET it be requir'd to draw a line by the point C, which ſhall be parallel to the line AB. Draw the line CE, and make the angle ECD equal to the angle CEA. I ſay the line CD is parallel to AB. *Demonſtration.*

The alternate angles DCE and CEA are equal: therefore the lines CD and AB are parallels.

'The eleventh Maxime, i. e. *If a line falling 'upon two others makes the internal angles leſs than 'two right angles, thoſe lines will concur*, may alſo 'now be eaſily demonſtrated.



Let the line AC, falling upon the lines AB and CD, make the internal angles ACD, and CAB, leſs than two right angles: I ſay that the lines AB and CD will concur. Let the angles ACD and CAE be equal to two right angles: the lines AE and CD will be parallels

[*by*

[*by the* 28.] Take the line AB as long as you please, and by the point B draw EF parallel to CA. Then take the line EB ſo oft as is neceſſary, to make it reach lower than the line CD; as in the preſent figure I have taken it only twice; ſo that EB and BF are equal. By the point F draw a parallel FG equal to AE, and joyn the line GB. I ſay that the line ABG is only one line; and that therefore the line AB concurring in FG, if the line CD be continu'd, ſince it cannot cut its parallel FG, it will cut the line BG between B and G.

Demonſtration.

The triangles AEB and BFG have the ſides AE and FG, BE and BF, equal; as alſo the alternate angles AEB, and BFG, (*by the* 29.) therefore they are equal in all reſpects, (*by the* 4.) And the oppoſite angles ABE, and FBG, are equal; and by conſequence [*by the coroll. of the* 15.] AB and BG make but one right line.

The USE.

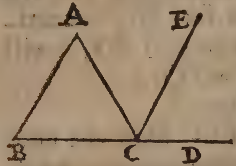
'The uſe of parallel lines is very common; 'as in *Perſpectives*, for as much as the appear-'ances or images of lines parallel to the picture 'or table, are parallel among themſelves. In '*Navigation*, the lines of the ſame Rhomb of 'the wind are deſcrib'd by Parallels. *Polar* '*Dials* have the hour-lines parallels. The Com-'paſs of Proportion is founded alſo upon parallels.

PROP

PROPOSITION XXXII.

A Theorem.

The external angle of a triangle is equal to both the internal opposite angles, taken together; and all the three angles of a triangle are equal to two right angles.



LET the side BC of the triangle ABC be produc'd to D: I say, that the external angle ACD is equal to both the internal angles A and B taken together. By the point C draw the line CE parallel to the line AB.

Demonstration.

The lines AB and CE are parallels, therefore [*by the* 29.] the alternate angles ECA and CAB are equal; and [*by the same*] the external angle ECD is equal to the internal B. And by consequence the whole angle ACD, being equal to both the angles ACE, and ECD, of which it is compos'd, will be equal to both the angles A and B taken together.

In the second place. The angles ACD and ACB are equal to two right angles, (*by the* 13.) and I have demonstrated the angle ACD to be equal to both the angles A and B taken together;

ther; therefore the angles ACB, A, and B, that is to ſay, all the angles of the triangle ABC, are equal to two right angles, or, which is all one, to 180 degrees.

Corollary 1. All the three angles of one triangle are equal to all the three angles of another triangle,

Coroll. 2. If two angles of one triangle be equal to two angles of another triangle, their third angles are alſo equal.

Coroll. 3. If a triangle has one right angle, the other two will be acute; and taken together will be equal to one right angle.

Coroll. 4. From a point given onely one perpendicular can be drawn to the ſame line; becauſe a triangle cannot have two right angles.

Coroll. 5. A perpendicular is the ſhorteſt of all the lines, that can be drawn from the ſame point to the ſame line.

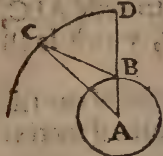
Coroll. 6. In a rectangle triangle the right angle is the greateſt angle, and the ſide oppos'd to it the greateſt ſide.

Coroll. 7. Every angle of an equilateral triangle contains 60 degrees; that is to ſay, the third part of 180.

The USE.

'This Propoſition is of uſe in *Aſtronomy* to 'determine the *Parallax.* Suppoſe the point

A

‘A to be the center of the earth; and that from


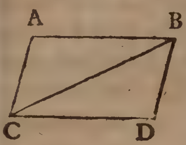
‘the point B upon the ſuperfi-
‘cies be taken the angle DBC,
‘that is to ſay, the diſtance of a
‘ſtar from the *Zenith* D. If the
‘earth was tranſparent the ſtar
‘would appear remote from the
‘*Zenith* D, according to the bigneſs of the an-
‘gle CAD, which is leſs than the angle CBD.
‘For the angle CBD being an external angle
‘in reſpect of the triangle ABC, it is (*by the*
‘32.) equal to both the oppoſite angles A and
‘C. Therefore the angle C will be equal to
‘the exceſs of the angle CBD above the angle
‘A. Whence I infer, that if I can know by
‘the *Aſtronomical tables* how far remote from
‘the *Zenith* the ſtar ought to appear to him
‘that ſhould be at the center of the earth, and
‘obſerve it at the ſame time *from the ſuperficies*,
‘the difference of thoſe two angles will be the
‘*Parallax* BCA.

PROP.

PROPOSITION XXXIII.

A THEOREM.

Two lines drawn towards the same parts, from the extremity of two other lines that are equal and parallel, are also themselves equal and parallel.



LET the lines AB and CD be parallel and equal; and let the lines AC and BD be drawn from their extremities towards the same parts; I say, that the lines AC and BD are equal and parallel. Draw the Diagonal line BC.

Demonstration.

Since the lines AB and CD are parallel, the the alternate angles ABC and BCD will be equal(*by the* 29.) therefore in the triangles ABC and BCD, which have the side BC common, and the sides AB and CD equal, together with the angles ABC and BCD equal also, the bases AC and BD will be equal, (*by the* 4.) and also the angles DBC, and BCA; which being alternate angles, the lines AC and BD will be parallel, (*by the* 27.)

The USE.

'This Proposition is reduc'd to practice for

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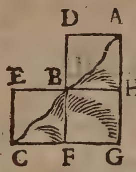


'the measuring the perpendicu-'lar hights, AG, of the vaſteſt 'Mountains; and alſo their hori-'zontal lines, CG, which are hid 'by their bulk. Take a large ſquare 'ADB, and place it ſo at the point 'A, that the ſide DB may fall per-'pendicularly; then meaſure the ſides AD 'and DB. This done, do the ſame again at the 'point B, and meaſure BE and EC: the ſides 'parallel to the horizon, AD, BE, added toge-'ther give the horizontal line CG; and the 'perpendicular ſides DB and EC, give the per-'pendicular hight AG. This way of meaſur-'ing is called * *Cultellation.*

* *Meaſuring by piece-meal.*

PROPOSITION XXXIV.

A THEOREM.

The oppoſite ſides and angles of a Parallelogram are equal; and the diameter divides it into two equal parts.

SUppoſe the figure ABDC [*ſee the fig. of the preceding Prop.*] to be a Parallelogram, that is to ſay, that the ſides AB, CD; AC, and BD, are parallel. I ſay, the oppoſite ſides AB, CD; AC, and BD, are equal; as alſo that the angles

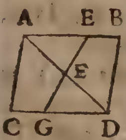
A

A and D, ABD, and ACD; and that the diameter BC equally divides the whole figure.

Demonstration.

The lines AB, and CD, are suppos'd to be parallels: therefore the alternate angles ABC and BCD will be equal, *(by the 29.)* In like manner the sides AC and BD being suppos'd to be parallels, the alternate angles ACB and CBD will be equal. And further, the triangles ABC, BCD, having the same side BC; and the angles ABC, BCD; ACB, and CBD equal, will be equal in all respects, *(by the 26.)* Therefore the sides AB, CD; AC, and BD, and the angles A and D, are equal: and the diameter divides the figure into two equal parts. And since the angles ABC, BCD; ACB, and CBD, are equal, joyning together ABC and CBD; and likewise BCD and ACB, we infer that the opposite angles ABD, and ACD are equal.

The USE.



'Surveyors have need of this Proposition for dividing grounds. If 'the field be a Parallelogram, they 'can divide it into two equal parts 'by the diameter AD. But if you 'be oblig'd to divide it by the point E: divide 'first the diameter into two equal parts by the 'point F, then draw the line EFG, which will 'divide the figure into two equal parts. For

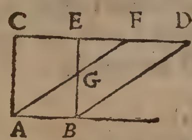
 the

'the triangles AEF and GFD, having the alter-
'nate angles EAF, FDG; and AEF, FGD;
'and the sides *AF* and *FD* equal, are equal,
'(*by the* 26.) And since the Trapezium BEFD
'with the triangle AEF; that is to say, the tri-
'angle ADB, is half the parallelogram, (*by the*
'34.) the same trapezium BEFD with the tri-
'angle DGF will be half the same. Therefore
'the line EG divides it in the middle.

PROPOSITION XXXV.

A THEOREM.

Parallelograms, having the same base, and being between the same parallels, are equal.



LET the Parallelograms be ABEC, and ABDF, having the same base AB, and being between the same parallels AB and CD. I say, they are equal.

Demonstration.

The sides AB, CE, are equal, (*by the* 34.) as also AB, *FD*: therefore CE and *FD* are equal; and adding to them *EF*, the lines *CF* and ED will be equal. The triangles therefore C*F*A, and EDB, have the sides CA, EB, as also *CF*, and ED equal, together with the angles DEB, and FCA, (*by the* 29.) one being an external, and the

the other an internal angle on the same side. Therefore (*by the* 4.) the triangle ACF and BED are equal; and taking from them both, that which is common, *viz.* the little triangle EGF, the trapezium FGBD will be equal to the trapezium CAGE: and adding to both the triangle AGB, the Parallelograms ABEC and ABDF will be equal.

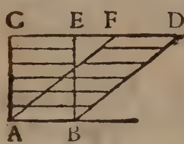
The USE.

' *Scotus*, and some Divines since him, have ' made use of this Proposition to prove, that ' *Angels may extend themselves to what space they* ' *please*. For supposing they can assume any ' figure, provided they have not a greater ex- ' tension: it is evident, that if an Angel should ' possess the space of the Parallelogram ABEC, ' it may likewise occupy the space of the Paral- ' lelogram ABDF; and because parallels may ' be continu'd *in infinitum*, (without end,) and ' Parallelograms may be still form'd longer and ' longer, which will all be equal to ABEC; an ' Angel will be able to extend it self still farther ' and farther.

A Demonstration of the same Proposition by Indivisibles.

' This method was lately invented by *Cava-* ' *lerius*; which has found different acception in the

'the world, ſome approving, and others reject-'ing it. His method conſiſts in this; that we 'imagine ſuperficies's to be compos'd of lines, 'like ſo many threds. And tis certain, that 'two pieces of linnen will be equal, if they have 'both the ſame number of threds, of equal 'length, and equally compacted.



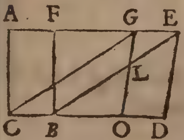
'Let two Parallelograms therefore ABEC, and ABDF, be propos'd, having the ſame baſe AB, and being between the ſame parallels AB, CD. Divide the parallelogram ABEC into 'as many lines as you pleaſe, parallel to AB, 'which continue to the other parallelogram 'ABDF. Tis evident there will be no more in 'one, than in the other; and that they will be 'of equal length, being all equal to the baſe 'AB; and that they will not be more cloſely 'compacted in one, than in the other: there-'fore the parallelograms will be equal.

PROP.

PROPOSITION XXXVI.

A THEOREM.

Parallelograms, upon equal bases, and between the same parallels, are equal.



LET the bases CB and OD of the parallelograms AC BF, ODEG, be equal; and let both be between the same parallels AE, CD. I say the parallelograms are equal. Draw the lines CG, and BE.

Demonstration.

The bases CB, and OD, are equal: OD, and GE, are also equal: therefore CB and GE are equal, and parallel; and by consequence (*according to the* 33.) CG and BE will be equal and parallel; and CBEG will be a parallelogram equal to CBFA, (*by the* 35.) having both the same base. In like manner, taking GE for the base, the parallelograms GODE and CBEG will be equal, (*by the same.*) Therefore the parallelograms ACBF, and ODEG, are equal.

The USE.

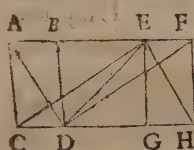
'We oft reduce parallelograms, which have 'oblique angles, as CBEG, or ODEG, to re-'ctangles; as CBFA: so that measuring the 'latter,

‘latter, which is easie, being only to multiply ‘AC by CB, the product being equal to the ‘parallelogram ACEF, we may by consequence ‘know the other parallelograms CBEG, or ‘ODEG.

PROPOSITION XXXVII.

A Theorem.

Triangles having the same base, and being between the same parallels, are equal.



If the triangles ACD and CDE, have the same base CD, and be inclos'd between the same parallels AF, and CH, they will be equal. Draw the lines DB, and DF, parallel to the lines AC, and CE, and you will have form'd two parallelograms.

Demonstration.

The Parallelograms ACDB, and ECDF, are equal (*by the* 35.) and the triangles ACD, CDE, are the halfs of those Parallelograms, (*by the* 34.) Therefore the triangles ACD, CDE, are equal.

PROP.

PROPOSITION XXXVIII.

A THEOREM.

Triangles, that have equal bases, and are inclos'd within the same parallels, are equal.

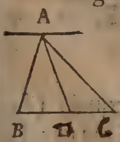
IF the triangles ACD, and ECH, [*see fig. preced.*] have equal bases CD, and GH, and are inclos'd within the same parallels AF, and CH, they are equal. Draw the lines BD and HF parallel to the sides AC, and EG; and you will have form'd two parallelograms.

Demonstration.

The Parallelograms ACDB, and EGHF, are equal, (*by the* 36.) and the triangles ACD and EGH are the halfs of those parallelograms, (*by the* 34.) therefore they are also equal.

The USE.

'We have in these propositions directions for 'dividing a triangular field into two equal parts; 'for example the triangle ABC. 'Divide the line which you will 'take for the base, as BC, into two 'equal parts in D: I say the triangles ABD, and ADC, are equal.



'For if you suppose a line drawn by A, parallel 'to BC, those triangles will have equal bases,

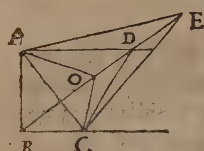
 'and

‘ and be inclos'd within the ſame parallels, and ‘ by conſequence will be equal. Other Diviſi- ‘ ons, grounded upon the ſame propoſition, ‘ might be made; but I omit them, that I ‘ might not be tedious.

PROPOSITION XXXIX.

A THEOREM.

Equal triangles, upon the ſame baſe, are within the ſame parallels.



IF the triangles ABC, and DBC, having the ſame baſe BC, be equal; the line AD drawn by the tops will be parallel to the baſe. For if AD and BC be no parallel; if you draw a parallel by the point A, it will fall either below the line AD, as AO; or above it, as AE. Suppoſe it to fall above, and produce BD till it meet the line AE, at the point E; then draw the line CE.

Demonſtration.

The triangles ABC and EBC are equal, (*by the* 38.) ſince the lines AE and BC are parallel; 'tis likewiſe ſuppos'd that the triangles ABC, and BDC, are equal: therefore the triangles

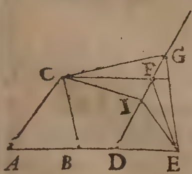
angles DBC and EBC would be equal; which is impossible, the first being part of the second. Whence I conclude, that a line parallel to BC cannot be drawn above AD, as AE.

I add, that that parallel cannot be below AD, as AO: because the triangle BOC would be equal to the triangle ABC, and by consequence to the triangle DBC; that is to say, the part would be equal to the whole. It must therefore be confess'd, that the line AD is parallel to the line BC.

PROPOSITION XL.

A Theorem.

Equal triangles, having equal bases, if they be taken upon the same line, are between the same parallels.



If the equal triangles ABC and DEF, have equal bases AB and DE, taken upon the same line AE; the line CF drawn by their tops will be parallel to AE. For if it be not parallel, having drawn by the point C a line parallel to AE, it will pass either above CF, as CG; or below it, as CI.

Demonstration.

If it pass above CF, as CG, continue DF till

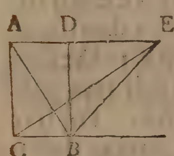
 it

it meet with CG in G; and draw the line EG. The triangles ABC and DGE would be equal, (*by the 38.*) and ABC and DEF being suppos'd to be equal, DEF, DGE, would be also equal; which, one being part of the other, cannot possibly be: therefore the parallel cannot pass above CF. I add, that neither can it pass below it, as CI; because then the triangles ABC and DEI would be equal, and by consequence DEI, and DEF; the part and the whole. Therefore only CF can be parallel to AE.

PROPOSITION XLI.

A THEOREM

A parallelogram will be double to a triangle, if they be between the same parallels, and have equal bases.



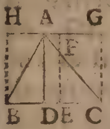
IF the parallelogram ACBD, and the triangle EBC, be between the same parallels AE and BC; and have the same base BC, or only equal bases; the parallelogram will be double the triangle. Draw the line AC.

Demonstration.

The triangles ABC and BCE are equal, (*by the*

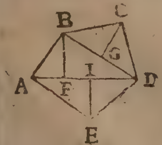
the 27.) But the parallelogram ACBD is double the triangle ABC, (*by the* 34.) It is therefore double the triangle BCE. It would be also double a triangle, that having a base equal to BC, should be between the same parallels.

The USE.



'The ordinary method of measuring the *area* or superficies of a triangle is built upon this proposition. If 'the triangle ABC be propos'd; from 'the angle A we must draw AD perpendicular to the base BC; then multiplying 'the perpendicular AD by half the base BE, the 'product gives the *area* of the triangle: because 'multiplying AD, or, what is the same, EF by 'BE, we have a rectangle BEFH, which is equal to the triangle ABC. For (*by the* 41.) 'the triangle ABC is half the rectangle HBCG; and so likewise is the rectangle BEFH.

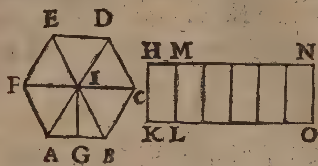
'We measure all sorts of rectilineal figures,



'as ABCDE, by dividing them into triangles, as BCD, ABD, 'AED; drawing the lines AD, 'and BD; and the perpendiculars CG, BF, and EI. For 'multiplying half of BD by CG, and half of 'AD by BF, and by EI, we have the *area* of all 'those triangles: adding which together the 'summe is equal to the rectilineal figure ABCDE.

We

'We find the *area* of regular Polygones, by 'multiplying half their circuit by a perpendi-'cular drawn 'from their cen-'ter to one of 'their ſides. For 'multiplying A 'G by IG, we



'ſhall have a rectangle HKLM equal to the 'triangle AIB : and repeating the ſame for all 'the other triangles, taking always half of the 'baſes, we ſhall have a rectangle HKON, which 'will have the ſide KO compounded of all the 'half baſes, and by conſequence equal to half 'the circumference; and the ſide HK equal to 'the perpendicular IG.

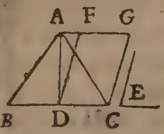
'Tis according to this principle, that *Archi-'medes* has demonſtrated, that a circle is equal 'to a rectangle compris'd under the ſemidia-'meter, and a line equal to half the circum-ference.

PROP.

PROPOSITION XLII.

A PROBLEM.

To make a Parallelogram equal to a triangle given, having one angle equal to an angle given.



LET a parallelogram be desir'd, equal to the triangle ABC, and having an angle equal to the angle E, divide the base BC into two equal parts at the point D; and draw the line AG parallel to BC, (*by the* 31.) then make the angle CDF equal to E, (*by the* 23.) and laſtly, draw the line CG parallel to DF: the figure FDCG is a parallelogram, becauſe the lines FG, DC; FD, and GC, are parallels, and its angle CDF is equal to the angle E; and further, tis alſo equal to the triangle ABC.

Demonſtration.

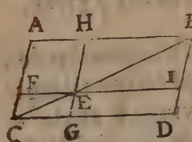
The triangle ADC is half the parallelogram FDCG, (*by the* 41.) 'tis alſo half the triangle ABC; ſince the triangles ADC, and ADB, are equal, (*by the* 38.) Therefore the triangle ABC is equal to the parallelogram FDCG.

PROP.

PROPOSITION XLIII.

A THEOREM.

The complements of a parallelogram are equal



IN the parallelogram ABDC, the complements AFEH, and FGDI, are equal.

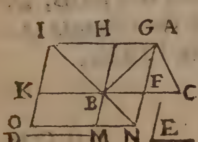
Demonstration.

The triangles ABC, and BCD, are equal, *(by the 34.)* therefore if the triangles HBE, and EIE, FEC, and CGE, which are also equal, *(by the same,)* be substracted, the complements AFEH, GDIE, which remain, will be equal.

PROPOSITION XLIV.

A PROBLEM.

To describe a parallelogram upon a line given, which shall be equal to a triangle, and have such a certain angle; i.e. equal to one given.



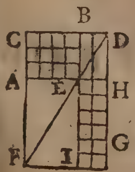
SUppose you be requir'd to make a parallelogram, which shall have one of its angles equal to the angle E, and one of its sides equal to the

the line D, and be equal to the triangle ABC. Make the Parallelogram BFGH, (*by the* 42.) which has the angle BFG equal to the angle E, and is equal to the triangle ABC. Produce the sides GF, and GH, so that HI may be equal to the line D; and draw the line IBN till it cuts GF produc'd to N; and from the point N draw the line NO parallel to GI, and IO parallel to BH; producing also the side FB to K, and HB to M. The parallelogram MK is that which you desire.

Demonstration.

GF and HM being parallels, the alternate angles GFB or the angle E, and FBM, are equal (*by the* 29.) In like manner the lines KB and MN being parallel, the alternate angles FBM, and BMO, are equal; therefore the angle BMO is equal to E, and the side KB is equal to the line HI, or D: and lastly, the parallelogram MK is equal to the parallelogram GFBH, (*by the preceding*,) and that was made equal to the triangle ABC. Therefore the parallelogram MK is equal to the triangle ABC.

The USE.



This Proposition contains a kind 'of *Geometrical* Division: for in '*Arithmetical* Division a number 'is propos'd, which may be lookt 'on as a rectangle: for example 'the rectangle AB, consisting of

'twelve ſquare feet, which is to be divided by
' another number, ſuppoſe, two; that is to ſay,
' another rectangle is deſir'd to be made equal
' to AB, having one of its ſides, BD, equal to
' two; and the queſtion is, how many feet the
' other ſide ought to contain; which is, as it were,
' the quotient. This is done Geometrically
' by the Rule and Compaſs. Take BD conſiſt-
' ing of two feet, and draw the Diagonal DEF:
' the line AF is that which you ſeek. For, hav-
' ing completed the rectangle DCFG, the com-
' plements EG, and EC, are equal, (*by the* 43,)
' and EG has for one of its ſides EH, equal to
' BD, of two feet in length; and EI equal to
' AF. This kind of Diviſion is call'd *Applica-*
' *tion*, becauſe the rectangle AB is apply'd to
' the line BD, or EH: and from hence tis, that
' all Diviſion is frequently call'd *Application*;
' becauſe the ancient *Geometricians* made more
' uſe of the Rule and Compaſs, than of *Arith-*
' *metick*.

PROP.

PROPOSITION XLV.

A PROBLEM.

To describe a parallelogram, whith shall have a certain angle; and be equal to a rectilineal figure given.

LET the rectilineal figure propos'd be ABCD, to which you are required to make an equal parallelogram, which shall have an angle equal to the angle E. Divide the rectilineal into triangles by the line BD: and (*by the* 42.) make a parallelogram FGHI, which has the angle IHG equal to the angle E, and is equal to the triangle ABD; and (*by the* 44.) make the parallelogram IHKL equal to the triangle BCD, having one side equal to IH, and the angle LIH equal to the angle E. The parallelogram FGKL will be equal to the rectilineal ABCD.

Demonstration.

*N*othing need be prov'd, but that the parallelograms FGHI, and HKLI, make up but one; that is to say, GH, and HK, make but one right line. The angles GHI, and LKH, are equal to the angle E. And the angles LKH,

 and

£ 29 and KHI, are equal to two right angles, because KHIL is a parallelogram. Therefore the angles GHI and KHI are equal to two right angles, and (*by the* 14.) GH and HK make one right line.

The USE.

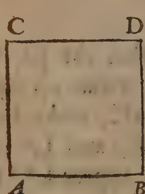
'The use of this proposition is the same with 'the preceding; serving to measure the capa'city of any figure whatsoever, by reducing it 'into triangles, and then making a rectangular 'parallelogram equal to them.

'Tis easie likewise to make a rectangular pa'rallelogram upon a determinate side, which 'may be equal to many irregular figures. In 'like manner having many figures a Rectangle 'may be described equal to their difference.

PROPOSITION XLVI.

A PROBLEM.

To describe a square upon a line given.



TO describe a square upon the line AB, draw two perpendiculars AC and BD equal to AB, and draw the line CD.

Demonstration.

The angles A and B being right angles,

angles, the lines AC and BD are parallels (*by the* 28.) They are alſo equal; therefore the lines AB and CD are parallels and equals, [*by the* 33.] and the angles A and C equal to two right angles; as alſo B and D, [*by the* 29.] and ſince A and B are both right angles, the angles C and D will be ſo likewiſe. Therefore the figure AD has all its ſides equal, and all its angles right angles, and by conſequence is a ſquare.

PROPOSITION XLVII.

A THEOREM.

The ſquare of the baſe of a rectangular triangle, is equal to the ſquares of both the other ſides taken together.

SUppoſe the angle BAC to be a right angle, and that ſquares were deſcribed upon all the ſides BC, AB, and AC: that of the baſe BC, which is oppos'd to the right angle, will be equal to the ſquares of both the ſides AB, and AC. Draw the line AH parallel to BD, and CE; and joyn the lines AD, AE, FC, and BG. I will prove the ſquare AF is equal to the rectangle BH, and the ſquare AG to the rectangle

gle CH; and therefore the ſquare BE is equal to both the ſquares AF and AG.

Demonſtration.

The triangles FBC, and ABD, have the ſides AB, BF; BD, and BC, equal: and the angles FBC, and ABD, are equal, each containing the angle ABC more than their reſpective right angles. Therefore [*by the* 4.] the triangles ABD and FBC are equal. But the ſquare AF is double the rriangle FBC, [*by the* 41,] having the ſame baſe BF, and being between the ſame parallels BF, and AC. In like manner the rectangle BH is double the triangle ABD, having likewiſe the ſame baſe BD, and being between the ſame parallels BD and AH. Therefore the ſquare AF is equal to the triangle BH. By the ſame method the triangles ACE, and GCB, may be prov'd to be equal, [*by the* 4.] and the ſquare AG to be double the triangle GCB; and the rectangle CH, double the triangle ACE, [*by the* 41.] therefore the ſquare AG is equal to the rectangle CH; and by conſequence the ſquares AF and AG are equal to the ſquare BDEC.

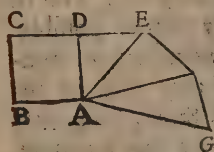
The USE.

'’Tis ſaid that *Pythagoras*, having found out 'this Propoſition, ſacrific'd a *Hecatomb*, i. e. a 'hundred Oxen, to the *Muſes*, to return them 'thanks for their aſſiſtance; ſuppoſing it, it ſeems,

'seems, above the power of bare humane in-'vention. Nor was his esteem thereof so irra-'tional, as to some perhaps it may appear; this 'proposition being the foundation of a very con-'siderable part of the *Mathematicks*. For in 'the first place *Trigonometry* cannot possibly 'subsist without it, it being necessary to com-'pose a table of all the lines that may be in-'scrib'd in a circle, as Chords, Sines, Tangents, 'Secants; as may appear by one example.

'Suppose the semidiameter AC to 'be divided into 100000 parts, & 'that the arch BC contains 30 'degrees. Since the chord, or line 'that subtends 60 degrees is equal 'to the semidiameter AC; BD the 'sine of 30 degrees, will be equal to half AC, 'and therefore contain 50000 parts. Now in 'the rectangular triangle ADB, the square of 'AB is equal to the squares of AD, and BD. 'Make therefore the square of AB, by multi-'plying 100000 by 100000, and from the pro-'duct substract the square of 50000 or BD; 'the remainder will be the square of AD, or 'BF the sine of the complement: and extract-'ing the square root of that number, you will 'have the line FB. This done, making as AD 'to BD, so AC to CE, you will have the tangent 'CE; then adding together the squares of 'AC and CE, the product (*by the 47.*) will give the

'the square of AE; extracting therefore from 'that number the square root, you will know 'the length of the line AE, which is the se-'cant.



'By this also we may aug-'ment figures, as much as 'we please. For example; 'to double the square ABC 'D, continue the side CD, 'so that AD and DE may 'be equal: the square of 'AE will be double the square ABCD; since '(*by the* 47.) it is equal to both the squares of 'AD and DE. Making the right angle AEF, 'and taking EF equal to AB, the square of AF 'will be triple the square ABCD. Again, 'making the right angle AFG, and taking FG 'equal to AB, the square of AG will be qua-'druple, or four times the square of ABCD. 'And that which I say of the square, may be 'understood of all similar figures.

PROP.

PROPOSITION XLVIII.

A THEOREM.

If in a triangle the square of one side be equal to the squares of both the other sides, taken together; the angle opposite to that first side will be a right angle.

IF the square of the side NP be equal to both the squares of the sides NL, and LP, taken together; the angle NLP will be a right angle. Draw LR perpendicular to NL, and equal to LP; then draw the line NR.

Demonstration.

In the rectangular triangle NLR, the square of NR is equal to the squares NL, and RL, or LP, (*by the* 47.) Now the square of NP is also equal to the same squares of NL and LP; therefore the square of NR is equal to the square of NP, and by consequence the lines NR and NP are equal. And because the triangles NLR, and NLP, have the side NL common; the sides LP and LR equal, and their bases NP and NR also equal; the angles NLP and NLR will be equal, (*by the* 8.) and the angle NLR being a right angle, the angle NLP must be so too.

N *The*

THE SECOND BOOK OF THE ELEMENTS OF EUCLID.

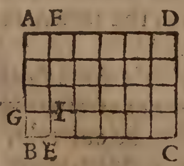
'EUCLID in this Book treats of the 'powers of right Lines; that is to say, 'of their Squares; comparing the di-'vers Rectangles, which are made upon a Line 'divided, as well with the Square, as the 'Rectangle, of the whole Line. 'Tis a part 'exceeding useful, serving for the foundation 'of the principal Operations of *Algebra*. The 'three first Propositions demonstrate the third 'Rule, or Operation of *Arithmetick*, *Multi-'plication*. The fourth teaches to extract the 'square Root of any number whatsoever. Those 'that follow to the Eight serve upon many oc-'casions in *Algebra*. The rest instruct us in O-'perations proper for *Trigonometry*. This Book 'seems at first view very difficult; because 'men are apt to imagine there is something my-'sterious contain'd therein; nevertheless the

'great-

'greatest part of its Demonstrations are grounded on this most evident Principle, that the 'whole is equal to all its parts taken together. 'But it ought not to discourage any, if they 'should not at the first attempt fully comprehend them.

DEFINITIONS.

1. *A rectangular parallelogram is compris'd under two lines, that form a right angle.*

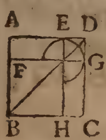


'OBserve that henceforward by a rectangle we 'shall intend such a parallelogram, whose angles are 'all right angles, distinguishing it by giving its longitude and latitude, naming two of its sides, 'which contain one of its angles, as the lines 'AB and BC. For the rectangle ABCD is 'compris'd under the lines AB and BC; BC 'denoting its longitude and AB its latitude; 'and the other being equal to these it will not 'be necessary to name them. I have also formerly intimated that the line AB, remaining perpendicular to BC, and being mov'd 'from one extremity thereof to the other, produces the rectangle ABCD; and that that

'motion has some resemblance to *Arithmetical*
'multiplication: so that, as the line AB mov-
'ing over the line BC, that is, taken so many
'times, as there are points in BC, composes
'the rectangle ABCD: so the multiplication
'also of AB by BC, will give the rectangle
'ABCD. As, suppose I knew the number of
'*Mathematical points*, that are in AB, for
'example 40, and that there were 60 in BC:
'it is evident that the rectangle ABCD will
'have so many lines equal to AB, as there are
'points in BC; and that multiplying 40 by
'60, the product will be 2400, which is the
'number of *Mathematical points* in the rectangle
'ABCD.

'I may take what quantity I please for a *Ma-*
'*thematical point*; provided I do not afterwards
'subdivide it; it must therefore be observ'd;
'that when I measure a line, for a *Mathemati-*
'*cal point* I take that measure which best suits
'with my occasions; for example, when I say
'a line of five foot in length, my *Mathema-*
'*tical point* is a line of a foot long; which I
'take without considering that it is compos'd
'of any parts. In measuring a superficies like-
'wise I do the same, taking some known super-
'ficies, for example, a foot square; which I do
'not afterwards subdivide. I make use of a
'square rather than any other figure; because
'its length and bredth being equal there is no
'need

'need of naming more than one of its dimensi-'ons to describe it. Accordingly when I would 'mark out the *Area* of the rectangle ABCD, 'I do not consider the sides as simple lines, but 'as rectangles of a determinate bredth: for 'example, when I say that the rectangle ABCD 'has the side AB of four foot long, since a foot 'is to me instead of a *Mathematical point*, I 'conceive the side AB to have also a foot in 'bredth, and to be as the rectangle ABEF. 'Therefore knowing how many times the 'bredth BE is contain'd in the line BC, I 'shall know how many times the line AB is 'contain'd in the rectangle ABCD; that is to 'say, multiplying AB which has four foot 'square by 6, the product will be 24 foot square. 'In like manner knowing the magnitude of the 'rectangle ABCD to be 24 foot square, and one 'of its sides AB to be 4; dividing 24 by 4 'the quotient will give me the other side BC, 'consisting of six foot square.



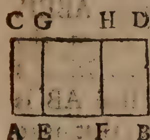
2. Having drawn the diameter of a rectangle, one of the lesser rectangles thro which it passes, together with the two complements, is call'd the Gnomon. 'As the rectangle EG, 'thro which the diameter BD passes, 'together with the complements EF and GH, 'is call'd the Gnomon; their figure together 'representing a Carpenters square.

PROP.

PROPOSITION I.

A Theorem.

If two lines be propos'd, whereof one is divided into divers parts, the rectangle contain'd under those two lines is equal to the rectangles contain'd under the line which is not divided, and the parts of the line divided.



LET the lines propos'd be AB, and AC; and let AB be divided into as many parts as you please. The rectangle AD contain'd under the lines AB and AC, is equal to the rectangle AG contain'd under AC and AE; to the rectangle EH contain'd under EG equal to AC, and EF; and to the rectangle FD contain'd under FH equal to AC and FB.

Demonstration.

The rectangle AD is equal to all its parts taken together; which are the rectangles AG, EH, and FD; and no other. Therefore the rectangle AD is equal to the rectangles AG, EH, and FD taken together.

By Numbers.

This proposition holds true likewise in numbers. Suppose the line AC to be five foot long; AE two, EF four, FB three; and by consequence AB nine: the rectangle contain'd under

under AC five; and AB nine, that is to say, five times nine, which makes forty five, is equal to twice five or ten, four times five or twenty, and three times five, or fifteen; for ten, twenty, and fifteen, make forty five.

The USE.

A,	53
B,	8
C,	50,3
B,	8
D,	24
E,	400
F,	424

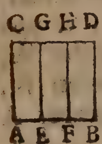
'By this proposition is demonstrated the ordinary operation of multiplication. For example, if you were to multiply the number A, which is 53, by the number B, that is 8. Divide the number A into so many parts as there are characters: that is, two, 50, and 3; which multiply by 8, saying, eight times three is twenty four; and so you make one rectangle. Then multiplying the number 50 by 8, the product will be 400. But 'tis evident that the product of eight times 53, being 424, is equal to the product of 24, and the product 400 taken together.

PROP.

PROPOSITION II.

A THEOREM.

The square of any line is equal to the rectangles contain'd under the whole line, and all its parts.



LET the line propos'd be AB, and its square ABDC. I say the square ABDC is equal to the rectangle contain'd under the whole line AB, and AE; another under AB and EF; and a third under AB and FB.

Demonstration.

The square ABDC is equal to all its parts taken together, which are the rectangles AG, EH, FD. The first AG is contain'd under AC equal to AB, and AE. The second EH is contain'd under EG equal to AC or AB, and FE. The third FD is contain'd under FH equal to AB, and FB: and 'tis the same thing to be contain'd under a line equal to AB, and to be contain'd under AB it self. Therefore the square of AB is equal to the rectangles contain'd under AB, and AE, EF, FB, the parts of AB.

By Numbers.

Let the line AB represent the number nine:

its square will be 81. Let also the part AE be four; EF three; and FB two: nine times four make thirty six; nine times three twenty seven; and nine times two eighteen; and 'tis plain that 36, 27, and 18 make 81.

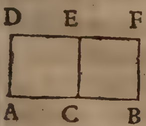
The USE.

'This proposition serves likewise to prove 'multiplication; as also for Equations in *Al-'gebra.*

PROPOSITION III.

A THEOREM.

If a line be divided into two parts, the rectangle contain'd under the whole line, and one of its parts, is equal to the square of the same part, and the rectangle contain'd under both the parts.



LET the line AB be divided into two parts at the point C; and let a rectangle be made of the whole AB and one of its parts AC, that is to say, let AD be equal to AC; and then if rectangle AF be compleated, it will be equal to the square of AC, and the rectangle contain'd under AC and CB. Draw the perpendicular CE.

Demonſtration.

The rectangle AF contain'd under AB and AD equal to AC, is equal to all its parts, which are the rectangles AE, and CF. The firſt AE is the ſquare of AC, the lines AC and AD being equal; and the rectangle CF is contain'd under CB, and CE equal to AD or AC. Therefore the rectangle contain'd under AB and AC is equal to the ſquare of AC, and the rectangle contain'd under AC and CB.

By Numbers.

Let AB be 8; AC 3; and CB 5: the rectangle contain'd under AB and AC, will be three times eight, or 24: the ſquare of AC 3, is nine; and the rectangle contain'd under AC 3, and CB 5, is three times 5 or 15. But it is evident that 15 and 9 make 24.

The USE.

A,	43	
C,	40,	3
B,		3
120.		9.
129		

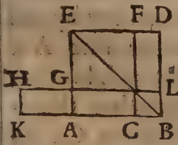
'The uſe of this propoſition is ſtill 'to demonſtrate the ordinary practiſe 'of multiplication. For example, if 'you would multiply the number 43 'by 3; having divided the number '43 into 40, and 3: three times 43 'will amount to as many as three times three, or 'nine, that is the ſquare of three; and three 'times 40, that is, 120; for three times forty 'three is 129. Beginners ought not to be diſ- 'cour-

'courag'd, if they do not presently apprehend 'these propositions; which yet, in truth are 'not difficult, but as they are conceiv'd to con- 'tain some strange mystery.

PROPOSITION IV.

A THEOREM.

If a line be divided into two parts, the square of the whole line, will be equal to the squares of both the parts; and two rectangles contain'd under the same parts.



LET the line AB be divided in C, and its square ABDE describ'd; let its diagonal also EB be drawn and a perpendicular cutting it CF: and by that point let the line GL be drawn parallel to AB. 'Tis evident that the square ABDE is equal to the four rectangles GF, CL, CG, and LF. The two first of which are the squares of AC and CB: and the two Complements are contain'd under AC and CB.

Demonstration.

The sides AE and AB are equal: therefore the angles AEB, and ABE are half right

angles: and becauſe the lines GL and AB are parallels; the angles of the triangles of the ſquare GF (*by the* 29. 1.) will be equal; as alſo their ſides (*by the* 6. 1.) Therefore GF is the ſquare of AC. In like manner CL is the ſquare of CB: the rectangle GC is contain'd under AC, and AG equal to BL or BC; and the rectangle LF is contain'd under LD equal to AC, and FD equal to BC.

Coroll. If you draw the diagonal of a ſquare the rectangles which it cuts are ſqares.

The USE.

A,	144
B,	22
C,	12

‘This propoſition teaches the method of extracting the ſquare root of ‘any number propos'd. Let the number be A, or 144, repreſented by the ‘ſquare AD, and its root by the line AB. ‘I ſuppoſe it known from other principles that ‘it requires two characters. I imagine therefore that the line AB is divided in C, ſo ‘that AC may repreſent the firſt character, ‘and BC the ſecond. Then ſearching the ‘root of the firſt character of the number 144, ‘which is 100, I find it to be 10: and making ‘its ſquare 100, repreſented by the ſquare GF, ‘I ſubſtract it from 144; and there remains 44 ‘for the rectangles GC, FL; and the ſquare ‘CL: But becauſe the figure of a Gnomon is ‘not proper for this operation, I tranſport the ‘rectangle FL unto KG, making one whole

‘rect-

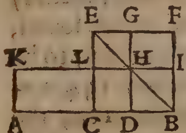
'rectangle KL, that is, 44. I know alſo al-
'ready almoſt the whole ſide KB: for AC be-
'ing 10, KC muſt be 20: I muſt therefore di-
'vide 44 by 20; that is to ſay, for my Di-
'viſor doubling the root found; I enquire then
'how many times 20 I can have in 44? and
'find twice; and therefore take 2 for the ſide
'BL; and becauſe 20 was not the intire ſide
'KB, but only KC; that two which came
'in the quotient I add to the Diviſor, making
'it 22; which number being found preciſely
'twice in 44, adding 2 to the root before found,
'I conclude the whole ſquare root of 144 to be
'12. You ſee then that the ſquare 144 is e-
'qual to the ſquare of 10, which is 100, the
'ſquare of 2, that is 4; and twice 20, which
'make the two rectangles contain'd under two
'and ten.

PROP.

PROPOSITION V.

A THEOREM.

If a line be divided into two equal parts and two parts that are unequal; the rectangle contain'd under the unequal parts, together with the square of the intermediate part; is equal to the square of half the line.



IF the line AB be divided into two equal parts in C; and two unequal parts in D; the rectangle AH contain'd under the unequal segments AD, and DB, with the square CD, will be equal to the square of CB, that is, the square CF. Compleat the figure as you see; the rectangles LG and DI will be squares. (*by the coroll. of the* 4.) I will prove then that the rectangle AH, contained under AD, and DH equal to DB, with the square LG, is equal to the square CF.

Demonstration.

The rectangle AL is equal to the rectangle DF, both being contain'd under half the line AB, and DB or DH, which is equal to it. Add to both the rectangle CH; the rectangle AH will be equal to the Gnomon CBG. Add there-

therefore again to both the ſquare LG; and the rectangle AH, with the ſquare LG will be equal to the ſquare CF.

By Numbers.

Let AB be 10; AC will be 5, and CB likewiſe; and let CD be 2 and DB 3: the rectangle contain'd under AD 7 and DB 3, that is to ſay 21, with the ſquare of CD 2, that is 4, will be equal to the ſquare CB 5, which is 25.

The USE.

'This propoſition is very uſeful in the third 'book: it is alſo us'd in *Algebra*, to demon-'ſtrate the manner of finding the root of an af-'fected or impure ſquare.

PROPOSITION VI.

A THEOREM.

If a line be divided into two equal parts, and to it another line added; the rectangle contain'd under the line compounded of thoſe two, and that which is added, with the ſquare of half the divided line, is equal to the ſquare of the line compounded of that half, and the line that is added.

IF to the line AB, divided into two equal parts in C, be added the line BD; The rectangle AN, contain'd under the line AD, and DN equal to BD, with the ſquare of

of CB, is equal to the ſquare of CD. Make the ſquare of CD, and having drawn the diagonal FD, draw alſo BG parallel to FC, cutting FD at the point H, through which paſſes the line HN parallel to AD. KG will be the ſquare of CB; and BN, that of BD.

Demonſtration.

The rectangles AK, and CH, being upon equal baſes AC and CB, are equal(*by the* 36.1.) The complements CH and HE are equal, (*by the* 43.1.) therefore the rectangles AK and HE are equal. Add to both the rectangle CN, and the ſquare KG: the rectangles AK and CN, that is, the rectangle AN, with the ſquare KG, will be equal to the rectangles CN and HE, and the ſquare KG, that is, the ſquare CE.

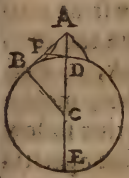
By Numbers.

Let AB conſiſt of 8 parts; AC of 4; and CB of 4; BD of 3. ſo that the whole AD be 11. Tis evident the rectangle AN is three times 11, that is 33; which with the ſquare of KG, equal to CB 4, that is 16; make 49, and therefore is equal to the ſquare of CD 7, which is 49; for 7 times 7 make 49.

The USE.

'*Maurylocus*, by the help of this Propoſition, 'meaſur'd the whole Earth at one ſingle Obſervation.

‘vation. To effect which, he advises, that ‘from the top of a mountain of ‘known hight A, you observe the ‘angle BAC, made by the line ‘AB, touching the superficies of ‘the earth at B, and the line AC ‘passing through the center: and ‘that in the triangle ADF, know-‘ing the angle A, and the right ‘angle ADF, you find by *Trigonometry* the sides ‘AF and FD: and because tis easie to demon-‘strate that FB and FD are equal, you will then ‘know the line AB, and also its square. Now ‘we have demonstrated in the preceding pro-‘position, that the line ED, being divided into ‘equal parts in C, and the line AD added to ‘it; the rectangle contain'd under EA, and ‘AD, with the square of CD, or CB, is equal ‘to the square of CA; and the angle ABC, ‘being a right angle, (as is prov'd in the third ‘book) the square of CA is equal to the squares ‘of AB and BC; therefore the rectangle un-‘der AE and AD, with the square of BC, is ‘equal to the squares of AB and BC. Take ‘therefore from them both the square of BC, ‘and the rectangle under AE, and AD, will be ‘equal to the square of AB. Divide therefore ‘the known square of AB, by the hight of the ‘mountain AD, and the Quotient will be the ‘line AE; from which substracting the hight



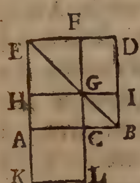
'of the mountain, the remainder will be the
'diameter of the earth DE.

'We have made use likewise of the same pro-
'position in our *Algebra*, to demonstrate the thir-
'teenth proposition of the third Book, to find
'the root of a square equal to a number, more a
'certain number of roots. The two that follow
'do also serve for the proof of the like operati-
'ons.

PROPOSITION VII.

A THEOREM.

If a line be divided, the square of the whole line, with that of one of its parts, is equal to two rectangles contain'd under the whole line, and that first part, together with the square of the other part.



LET the line AB be divided any where in C; the square AD of the line AB, with the square AL, will be equal to two right angles contain'd under AB and AC, with the square of CB. Make the square of AB, and having drawn the diagonal EB, and the lines CF and HGI; prolong EA so far, as that AK may be equal to AC: so AL will be the square of AC, and HK will be equal to AB; For HA is equal

to GC, and GC is equal to CB, becauſe CI is the ſquare of CB, *(by the Corol. of the 4.)*

Demonſtration.

Tis evident, that the ſquares of AD and AL are equal to the rectangles HL and HD, and the ſquare CI. Now the rectangle HL is contain'd under HK equal to AB, and KL equal to AC. In like manner the rectangle HD is contain'd under HI equal to AB, and HE equal to AC. Therefore the ſquares of AB and AC are equal to two rectangles contain'd under AB and AC, and the ſquare of CB.

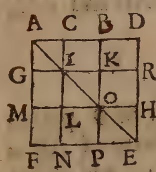
In Numbers.

Suppoſe the line AB to conſiſt of 9 parts, AC of 4, and CB of 5. The ſquare of AB 9 is 81, and that of AC 4 is 16; which 81 and 16 added together make 97. Now one rectangle under AB and AC, or 4 times 9, make 36, which taken twice is 72: and the ſquare of CB 5 is 25; which 72 and 25 added together make alſo 97.

PROPOSITION VIII.

A THEOREM.

If you divide a line, and add another to it equal to one of its parts, the square of the whole compounded line will be equal to four rectangles contain'd under the first line, and that part that is added, together with the square of the other part.



LET the line AB be divided any where in the point C, and BD equal to CB added to it: the square of AD will be equal to four rectangles contain'd under AB, and BC or BD, and the square of AC. Make the square of AD, and having drawn the diagonal AE, draw likewise the perpendiculars BP, and CN, cutting the diagonal in I, and O: and also the lines MOH, and GIR, parallel to AB. The rectangles GC, LK, PH, MB, and NR, will be squares, (*by the Coroll. of the* 4.)

Demonstration.

The square ADEF is equal to all its parts: and the rectangles LB, OD, PM, are contain'd under lines equal to AB, and BC, and if you add the rectangle MI to the rectangle PH, they together will give you another rectangle contain'd under

under a line equal to AB, and another equal to CB or BD. Besides which there remains nothing but the square GC, which is the square of AC. Therefore the square AD is equal to four rectangles contain'd under AB and BD, and the square of AC.

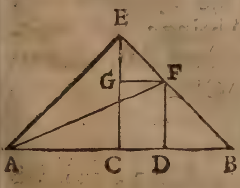
In Numbers.

Let the line AB consist of 7 parts, AC of 3, and CB of 4; as also BD: the square of AD 11 will be 121. And one rectangle under AB 7, and BD 4, makes 28; which taken four times is 112; and those together with the square of 3, which is 9, make also 121.

PROPOSITION IX.

A THEOREM.

If a line be divided into two equal parts, and two unequal, the squares of the unequal parts will be double the square of half the line, and the square of the intermediate part.



LET the line be divided into two equal parts at the point C, and two unequal at the point D: the squares of the unequal parts AD, and DB wil

will be double the squares of AC, which is half AB, and CD the intermediate part. Draw CE perpendicular to AB, and equal to AC; draw also the lines AE, and BE, and the perpendicular DF, as likewise FG parallel to CD. Then joyn the line AF.

Demonstration.

The lines AC and CE are equal, and the angle C is a right angle: therefore (*by the* 5. 1.) the angles CAE, and CEA, are equal; and consequently half right angles. In like manner, the angles CEB, CBF, GFE, and DFB, are half right angles; and the lines GF and GE, DF and DB, equal, (*by the* 6. 1.) and the whole angle AEF is a right angle. Now the square of AE (*by the* 47. 1.) is equal to the squares of AC and CE, which are equal: therefore it is double the square of AC. For the same reason, the square of EF is double the square of GF, or CD. Now the square of AF is equal to the squares of AE, and EF, because the angle AEF is a right angle: therefore the square of AF is double the squares of AC, and CD. The same square of AF is likewise equal to the squares of AD, and DF or DB, the angle D being a right angle. Therefore the squares of AD, and DB, are double the squares of AC, and CD.

In Numbers.

Let AB be 10, AC 5, CD 3, and DB 2:

2 : the squares of AD 8, and DB 2, that is to say, 64 and 4, which make 68, are double the squares of AC 5, that is, 25, and of CD 3, which is 9 : for 25 and 9 make 34, which is half of 68.

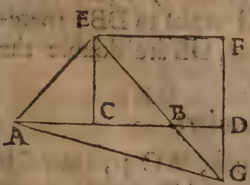
The USE.

"I have not met with this Proposition, except in *Algebra*; no more than that which follows.

PROPOSITION X.

A Theorem.

If a line be added to another that is divided into two equal parts; the square of the line compounded of those two, with the square of that which is added, makes double the square of half the line, and the square of that which is compounded of half, and the line that is added.



LET the line AB be divided in the middle at the point C, and the line BD added to it: the squares of AD, and BD, will be double the squares of AC, and CD. Draw the perpendiculars CE and DF equal to AC: and then draw the lines AE,

AE,EF; and producing FD to G, so that DG may be equal to BD, joyn the lines AG, and EBG.

Demonstration.

The lines AC,CB, and CE being equal, and the angles at the point C being right angles: the angles CAE, AEC, CEB, and CBE, will be half right angles. In like manner the angle D being a right angle, and the lines BD and DG equal, the angles DBG, and DGB, will be half right angles; and so will likewise GEF, the angle F being a right angle; therefore the lines FG and FE are equal, *(by the* 6. 1.) and EF is equal to CD, *(by the* 33. 1.) Now the square of AE is double the square of AC, and the square of EG also double the square of EF, or CD, *(by the* 47. 1.) But the square of AG is equal to the squares of AE and EG, *(by the same:)* therefore the square of AG is double the squares of AC and CD. The same square of AG is likewise *(by the same)* equal to the squares of AD, and DG equal to DB: therefore the squares of AD and DB are double the squares of AC and CD.

By Numbers.

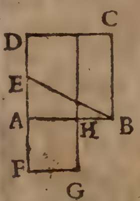
Let AB contain 6 parts, AC 3, and CB 3, BD 4; the square of AD 10 is 100; the square of BD 4 is 16, which make together 116. The square also of AC 3 is 9: the square

ſquare of CD 7 is 49. Now 49 and 9 make 58, the half of 116.

PROPOSITION XI.

A Problem.

To divide a line in ſuch a manner, that the rectangle under the whole line, and one of its parts, ſhall be equal to the ſquare of the other part.



Suppoſe the line AB to be divided in ſuch a manner, that the rectangle under the whole line AB, and BH, may be equal to the ſquare of AH. Make the ſquare of AB, (*by the* 46. 1.) and dividing AD in the middle in E, draw EB, and take EF equal to EB. Then make the ſquare of AF, that is to ſay, let AF and AH be equal. I ſay, the ſquare of AH will be equal to the rectangle HC, contain'd under HB, and BC equal to AB.

Demonſtration.

The line AD is divided equally in the point E, and the line FA added to it, therefore (*by the* 6.) the rectangle DG contain'd under DF, and FG equal to AF, with the ſquare of AE, is equal to the ſquare of EF, equal to EB : now the ſquare of EB is equal to the ſquares of AE and

AB, (*by the* 47.1.) therefore the ſquares of AB and AE are equal to the rectangle DG, and the ſquare of AE: and ſubſtracting from both the ſquare of AE; the ſquare of AB, that is, AC, will be equal to the rectangle DG: taking away therefore the rectangle DH, which is common to both, the rectangle HC will be equal to the ſquare of AH, that is, AG.

The USE.

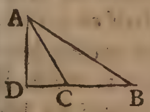
This propoſition teaches how to cut a line 'according to the extreme and middle propor-'tion, as will be ſhown in the *6th.* Book, 'Tis 'alſo frequently made uſe of in the 14*th.* book 'of *Euclid's Elements*, to find the ſides of regu-'lar Solids. It is uſeful alſo in the 11. *of the* '4. to inſcribe a Pentagone in a circle, as alſo a 'Pentedecagone (or a figure with 15. angles.) 'You will ſee alſo other uſes thereof in divid-'ing lines on this manner, in the 30*th.* propo-'ſition of the 6.

PROP.

PROPOSITION XII.

A THEOREM.

In an Obtusangle triangle, the square of the side oppos'd to the obtuse angle, is equal to the squares, of both the other sides, and two rectangles contain'd under the line upon which a perpendicular will fall, and the line which lies betwixt the triangle and the perpendicular.



LET the angle ACB, of the triangle ABC, be an obtuse, and let AD be drawn perpendicular to BC; the square of the side AB is equal to the squares of the sides AC and CB, and two rectangles contain'd under the side BC, and DC.

Demonstration.

The square of AB is equal to the squares of AD, and DB, (*by the* 47.1.) But the square of DB is equal to the squares DC, and CB, and two rectangles contain'd under DC and CB, (*by the* 4.) Therefore the square of AB is equal to the squares AD, DC, and CB, and two rectangles contain'd under DC and CB. In stead of the two first squares AD, and DC, put the square of AC, which is equal to them, (*by the* 47. 1.) The square AB will be equal to the squares

AC and CB, and two rectangles contain'd under DC and CB.

The USE.

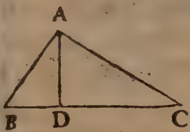
'This proposition is useful in *Trigonometry* 'to measure the *area* of a triangle, whose sides 'are known. For example, suppose the side 'AB to consist of twenty foot, AC of 13, BC 'of 11: the square of AB will be 400, that of 'AC 169, and that of BC 121. The summe 'of the two last is 290, which substracted from '400, there will remain 110 for the two rect-'angles under BC and CD. The half of which, '55, will make one half of those rectangles; 'dividing which number by BC, 11, we shall 'have 5 for the line CD; whose square 25 be-'ing substracted from the square of AC, 169, 'leaves the square of AD, 144, whose root 12 'will be the side AD; which being multiplied 'by $5\frac{1}{2}$, the half of BC, will give the *area* of 'the triangle ABC, that is, 66 foot square.

PROP.

PROPOSITION XIII.

A THEOREM.

In any triangle whatſoever, the ſquare of the ſide oppos'd to the acute angle, with two rectangles contain'd under the ſide upon which the perpendicular falls, and the line which is betwixt the perpendicular and that angle; is equal to the ſquares of both the other ſides.



SUppoſe the triangle to be ABC, and the acute angle C, and AD the perpendicular falling upon BC: the ſquare of the ſide AB, oppos'd to the acute angle C, with two rectangles contain'd under BC and CD, will be equal to the ſquares of AC, and BC.

Demonſtration.

The line BC being divided in D, *(by the 7.)* the ſquares of BC and DC are equal to two rectangles under BC and CD, and the ſquare of BD. Add to both the ſquare of AD: the ſquares of BC, DC, and AD, will be equal to two rectangles under BC, and CD, and the ſquares of BD, and AD. In ſtead of the ſquares of CD, and AD, put the ſquare of AC, which is equal to them, *(by the* 47. 1.) and inſtead of the

the squares of BD, and AD, substitute the square of AB, which is equal to them, (*by the same*:) the squares of BC, and AC, will be equal to the square of AB, and two rectangles contain'd under BC, and CD.

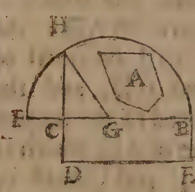
The USE.

'These propositions are very useful in *Trigonometry*: I have made use of them in the 'eighth proposition of my third book, to prove, 'that in a triangle the Sine total has the same 'proportion to the sine of an angle, as the rect-'angle contain'd under the sides, which form 'that angle, to double the triangle. I have 'us'd them likewise in the seventh, and divers 'other propositions.

PROPOSITION XIV.

A PROBLEM.

To describe a square equal to a rectilineal figure given,



TO describe a square equal to the rectilineal A, make (*by the* 45.1.) a rectangle BCDE equal to the rectilineal A. If the sides CD, and CB were equal, we should

ſhould have what we deſir'd : but being unequal, continue the line BC, ſo that CF may be equal to CD ; and dividing the line BF, in the middle at the point G, deſcribe the ſemicircle FHB; this done, prolong DC to H. The ſquare of CH is equal to the rectilineal A. Draw the line GH.

Demonſtration.

The line BF is divided into two equal parts in G, and two unequal in C: therefore (*by the* 5.) the rectangle contain'd under BC, CF, or CD, that is to ſay, the rectangle BD, with the ſquare of CG, is equal to the ſquare of GB, or GH, which is equal to it. Now (*by the* 47. 1.) the ſquare of GH is equal to the ſquares of CG, and CH: therefore the rectangle BD, and the ſquare of CG, are equal to the ſquares of CG, and CH ; and therefore taking away the ſquare of CG, which is common to both, there will remain the ſquare of CH equal to the rectangle BD, or, which is the ſame, the rectilineal A.

The USE.

'This propoſition teaches us in the firſt place 'to reduce any rectilineal figures to ſquares; 'which being the chief meaſure of all ſuperficies's, becauſe its dimenſions are both equal, 'we can by this means take the magnitude of 'all ſorts of rectilineal figures. Again it helps 'us to find a middle proportional betwixt two

lines

'lines given, as we ſhall ſee in the thirteenth
'propoſition of the ſixth book.

' *Ariſtotle* brings this propoſition as an in-
'ſtance of a Formal Definition: for, *in his ſecond*
'*book, de Anima, ſect.* 12. diſtinguiſhing be-
'twixt a Formal and a Cauſal Definition, he ex-
'plains them thus. If, when tis demanded
'What it is to ſquare a Rectangle? anſwer be
'return'd, that it is to deſcribe a ſquare equal
'to a rectineal; this anſwer contains the for-
'mal definition. But if it be ſaid, that it is to
'find a middle proportional betwixt two lines;
'this gives the Cauſal definition. For to find
' a middle proportional is the cauſe of making
'a ſquare equal to the rectilineal propos'd.

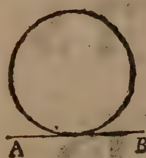
This propoſition may alſo be farther uſeful
'for the ſquaring of crooked figures; and alſo,
'as far as is poſſible, even the Circle it ſelf; for
'all ſorts of crooked figures may, at leaſt as far
'as is diſcernible by ſenſe, be reduc'd to recti-
'lineals. As for example, if we inſcribe in a
'circle a Polygone conſiſting of a thouſand
'ſides, there will be no ſenſible difference be-
'twixt it and the circle: therefore reducing
'this Polygone to a ſquare, we do, as far as our
'ſenſes are capable of judging, ſquare the circle.

THE

THE THIRD BOOK OF THE ELEMENTS OF EUCLID.

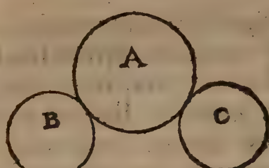
'THis third Book explains the properties of
'a Circle, and compares divers lines
which may be drawn within, or without
' its circumference. It considers likewise the
' circumstances of circles, that cut each other.
' or touch a right line; and the differences
' of angles that are made either at the centers
' or circumferences. In fine it lays down the
' first principles for the establishing the practical
' part of *Geometry*; for which the circle is
' most commodiously made use of in almost all
' Treatises of the *Mathematicks*,

DEFINITIONS.

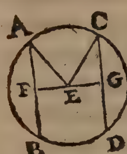


1. Thoſe Circles are equal, whoſe diameters or ſemidiameters are equal.

2. A line is ſaid to touch a circle, when, meeting with its circumference, it does not cut it. As the line AB.

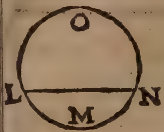


3. Circles touch, when meeting, they do not cut each other. As the circles A,B,and C.



4. Thoſe lines are equally remote from the center; when the perpendiculars, drawn from the center to the lines, are equal. "As for 'example, if EF, and EG, perpendiculars to the lines AB, and CD, 'be equal, AB and CD will be equally remote 'from the center; becauſe the diſtance ought 'always to be meaſur'd by perpendicular lines.

5. A



5. A ſegment of a circle is a figure terminated on one ſide by a right line, and on the other by the circumference of a circle. As LON, LMN.

6. The angle of the ſegment is the angle which the circumference makes with the right line. “ As the angles LNO, NLM.

7. An angle is in that ſegment in which are the lines that form it. ‘ As the angle FGH, is in the ſeg-‘ ment FGH.

8. An angle is upon that arch, to which it is oppos'd, or which is as its baſe. “ As the “ angle FGH, is upon the arch FIH.

9. The Sector is a figure contain'd under two ſemidiameters, and the arch which ſerves them for a baſe. ‘ As the figure FIGH.

PROPOSITION I.

A PROBLEM.

To find the center of a Circle.

TO find the center of the circle AEBD, draw the line AB, and divide it in the middle at the point C; through which draw the perpendicular ED, which also divide into two equal parts at the point F, and that point F will be the center of the circle. If it be not, suppose the point G to be the center; and draw the lines GA, GB, and GC.

Demonstration.

If the point G be the center, the triangles GAC, and GBC, will have the sides GA, and GB equal, (*by the definition of a circle*:) and AC and CB will be equal, the line AB being divided in the middle at the point C, and CG being common, the angles GCB, and GCA will be equal, (*by the* 8. 1.) and CG a perpendicular, not CD, which is contrary to the supposition. Therefore the center must of necessity be in the line CD. I add, that it must be at the point F, where it is divided into two equal parts: otherwise

wise the lines drawn from the center to the circumference would not be equal.

Coroll. The center of a circle is in that line, which falling perpendicularly upon another, divides it into two equal parts.

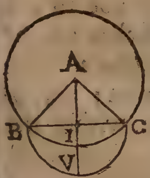
The USE.

"This proposition is necessary to demon-
"strate those that follow.

PROPOSITION II.

A THEOREM.

A right line drawn from one point of the circumference to another, falls wholly within the circle.



LET a line be drawn from the point B to the point C. I say, it will be wholly contain'd within the circle. To prove that it cannot fall without the circle, as BVC; having found the center of the circle A, draw the lines AB, AC, and AV.

Demonstration.

The sides AB, and AC, of the triangles ABC, are equal: therefore (*by the* 5. 1.) the angles ABC and ACB are equal, And since the angle AVC is an external angle in respect of the triangle

angle AVB, it is greater than the angle ABC, (*by the* 16. 1.) and then alſo it will be greater than the angle ACB. Therefore (*by the* 19.1.) in the triangle ACV, the ſide AC, oppos'd to the greater angle AVC, will be greater than AV: and by conſequence AV ought not to reach to the circumference of the circle, if the line BVC was a right line.

The USE.

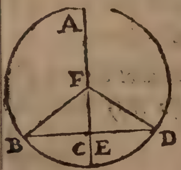
'Tis by this propoſition that they demon-'ſtrate, that a circle can touch a right line but 'in one place. For if the line touch'd two 'points of the circumference, it would be 'drawn from one of its points to another: and 'by conſequence, according to this propoſition, 'would enter the circle; though by its defini-'tion, the line that touches ought not to cut 'the circumference. *Theodoſius* makes uſe of 'the ſame Demonſtration to prove, that a Globe 'can touch a plane only in one point; for o-'therwiſe the plane would enter within the 'Globe.

PROP.

PROPOSITION III.

A Theorem

If the Diameter divide a line, which does not pass through the center, into two equal parts, it will cut it at right angles; and if it cut it at right angles, it will divide it into two equal parts.



IF the diameter AC, cut the line BD, which does not pass through the center *F*, into two equal parts at the point E, it will cut it at right angles. Draw the lines FB, and FD.

Demonstration.

In the triangles FEB, and FED, the side EF is common; the sides BE and ED are equal, because the line BD is equally divided in E, and their bases FB and FD are equal: therefore (*by the* 8.1,) the angles BEF and DEF are equal, and by consequence right angles. I add, that if the angles BEF and DEF be right angles, the line BD will be divided into two equal parts at E, that is to say, the lines BE and ED will be equal.

Demonstration.

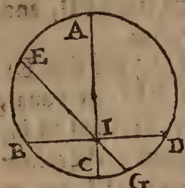
The triangles BEF and DEF are rectangular: therefore (*by the* 47. 1.) the square of the side

side DF will be equal to the squares of the sides ED, and EF. Now the squares of BF and FD are equal, because the lines are equal, therefore the squares of BE and EF are equal to the squares of DE and EF; and taking away the square of EF, the squares of BE, and ED will be equal, and by consequence the lines.

PROPOSITION IV.

A THEOREM.

Two lines drawn within a circle cannot cut each other into two equal parts, unless they both pass through the center.



IF the lines AC and BD cut each other at the point I, which is not the center of the circle, they will not equally divide each other. First, if one of those lines, as AC, pass through the center, tis evident it cannot equally be divided but at the center. But if neither pass through the center, as BD and EG, draw the line AIC through the center.

Demonstration.

If the line AC divide the line BD into two equal parts in I, the angles AID and AIB will be

be right angles, (*by the 3.*) In like manner if the line EG was equally divided in I, the angle AIE would be a right angle; and consequently the angle AIB and AIE would be equal, which is impossible, one being part of the other. In a word the line AIC, which passes through the center, would be perpendicular to the lines BD and EG, if they were both equally divided at the point I.

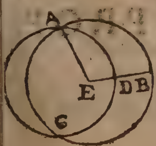
The USE.

These two Propositions are us'd in *Trigonometry*, to demonstrate, that the half of a chord of an arch is perpendicular to the semidiameter; and consequently, that it is the sine of half the arch. By these also they demonstrate, that the sides of a triangle have the same proportion, as the sines of the opposite angles. We also make use of it to find the Eccentricity of the Circle, which the Sun describes in his annual motion.

PROPOSITION V.

A THEOREM.

Circles that cut each other, have not the same center.



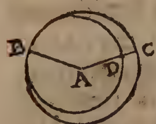
THE circles ABC, and ADC, which cut each other in A and C, have not the same center. If they had the same center,

ſuppoſe E, the lines EA and ED would be equal, (*by the definition of a circle*;) as alſo the lines EA, and EB: therefore the lines ED and EB would be equal, which is impoſſible, one being part of the other.

PROPOSITION VI.

A THEOREM.

Two circles that touch each other on the inner ſide have not the ſame center.



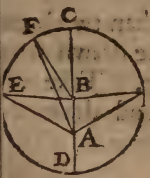
THE circles BD and BC, which touch each other on the inner ſide at the point B, have not the ſame center. For ſhould the point A be ſuppos'd to be the center of both the circles; the lines AB and AC, AB and AD, would be equal, (*by the definition of a circle*,) and conſequently the lines AD and AC would be equal, which is impoſſible, one being part of the other.

PROP.

PROPOSITION VII.

A THEOREM.

If many lines be drawn from any one point within the circle, which is not its center, to the circumference: 1. that which passes through the center is the greatest: 2. the remainder of it, continu'd to the opposite part of the circumference, is the least: 3. that which is nearest to the greatest, exceeds those that are more remote: 4. There can be no more than two of them equal to each other.



Suppose many lines to be drawn from the point A, being not the center of the circle, to the circumference; and the line AC to pass through the center B: I will demonstrate, that it is greater than any of the other; for example, that it is greater than AF. Draw FB.

Demonstration.

The sides AB and BF of the triangle ABF, are greater than AF alone, (*by the* 20.1.) But BF and BC are equal, (*by the definition of a circle*:) therefore AB and BC, that is to say, AC, is greater than AF.

I add in the second place, that AD is the

least; for example, that it is less than AE. Draw BE.

Demonstration.

The sides EA and AB are greater than BE alone, but BE is equal to BD, therefore EA and AB are greater than BD: taking therefore from both that which is common AB, AE will remain greater than AD.

Further, AF, which is nearer AC than AE, is also greater than it.

Demonstration.

The triangles FBA, and EBA, have the sides BF and BE equal, and BA is common to both: but the angle ABF is greater than the angle ABE: therefore (*by the* 24. 1.) AF is greater than AE.

Lastly, I say, that no more than two lines, that are equal to each other, can be drawn from the point A to the circumference. Take the angles ABE and ABG equal; and draw the lines AE and AG.

Demonstration.

The triangles ABG, and ABE, having the sides BE and BG equal; the side AB common to both, and the angles ABE and ABG equal; therefore their bases AE, and AG will be equal, (*by the* 4. 1.) But all the lines that can be drawn either on one side or the other, will be either nearer AC, than AE, and AG; or more remote from it; and accordingly will be either greater or less than AG. Therefore there can no more

more than two lines equal betwixt themſelves be drawn from the point A to the circumference.

The USE.

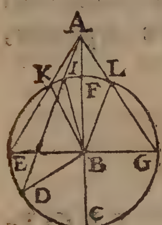
'*Theodoſius* advantageouſly uſes this propoſi-
'tion to prove, that if from any point of the ſu-
'perficies of a ſphere, which is not the pole of
'any certain circle, divers arches of greater
'circles be drawn to the circumference of that
'circle, that which paſſes through its pole will
'be the greateſt. For example: If from the pole
'of the world, which is diſtinct from the pole of
'the Horizon, (for the Zenith is its pole,) divers
'arches of greater circles be drawn to the cir-
'cumference; the arch of the Meridian, which
'paſſes through the Zenith, will be the greateſt
'arch. This propoſition is alſo brought to
'prove, that the Sun, when in his *Apogæum*,
'is moſt remote from the Earth.

PROP-

PROPOSITION VIII.

A THEOREM.

If from a point taken without the circle, many lines be drawn to its circumference, 1. of all those that extend to the concave circumference, that which passes through the center is the greatest: 2. those that lye nearest to it, are greater than those that are more remote: 3. among those that fall upon the convex circumference, that which being continu'd passes through the center, is the least: 4. the nearer to that are less than those farther off: 5. there can be but two equal lines drawn from the same point either to the concave or convex circumference.



Suppose many lines were drawn from the point A to the circumference of the circle GCDE.

First, the line AC, which passes through the center B, is the greatest of all those that reach to the concave circumference; for example, it is greater than AD. Draw the line BD.

Demonstration.

In the triangle ABD, the sides AB and BD are greater than AD alone; but the sides AB

and

and BC are equal to AB and BD: therefore AB and BC, or AC, is greater than AD.

2. AD is greater than AE.

Demonstration.

The triangle ABD and ABE, have the side AB common to both, and the sides BD and BE equal, and the angle ABD is greater than the angle ABE: therefore (*by the* 24. 1,) the base AD is greater than the base AE.

3. AF, which being continu'd passes through the center, is the least of all those that are drawn to the convex circumference LFIK; for example, it is less than AI. Draw IB.

Demonstration.

In the triangle AIB the sides AI and IB are greater than AB alone, (*by the* 20. 1.) therefore taking from both the equal lines BI and BF, AF will remain less than AI.

4. AI is less than AK. Draw the line BK.

Demonstration.

In the triangles AIB and AKB, the sides AK and KB are greater than the sides AI, and IB, (*by the* 21. 1.) therefore taking from both the equal sides BK, and BI, AI will remain less than AK,

5. There can be but two lines equal betwixt themselves drawn. Take the angles ABL, and ABK; as also ABE, and ABG, equal.

Demonstration

The triangles ABL, and ABK, will have their bases

bases AL and AK equal, (*by the* 4. 1.) and by the same also AE and AG will be equal; but no other line can be drawn, that will not be either nearer to, or more remote from AF, or AG; and consequently, that will not be either greater or less than AK and AL, AE and AG.

PROPOSITION IX.

A THEOREM.

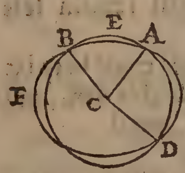
That point from whence three equal lines can be drawn to the circumference of a circle, is its center.

IF the point were not the center of a circle, there could be but two equal lines drawn from it to the circumference, (*by the* 7, and 8.)

PROPOSITION X.

A THEOREM.

Two circles cut each other only in two points.



IF two circles AEBD, and ABFD, should cut each other in three points A, B, and D; find (*by the* 1.) the center C of the circle AEBD; and draw the lines CA, CB, and CD. *De-*

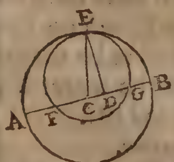
Demonstration.

The lines CA, CB, and CD, drawn from the center C to the circumference of the circle AEBD, are equal: but the same lines are also drawn to the circumference of the circle ABFD: therefore (*by the 9.*) the point C will be the center of the circle ABFD. So that two circles, which cut each other, will have the same center; which is contrary to the fifth proposition.

PROPOSITION XI.

A THEOREM.

If two circles touch each other on the inside, a line drawn through both the centers, will also pass through the point where they touch.



IF the two circles EAB and EFG touch each other on the inside, at the point E; a line drawn through both their centers will pass through the point E. For if the point D was the center of the lesser circle, and C that of the greater, so that the line CD passing through both should not pass through the point E: draw the lines CE and DE.

Demonstration.

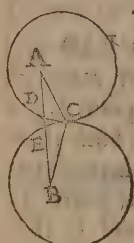
The lines DE, and DG, drawn from the

center of the lesser circle D to its circumference, would be equal: and adding the line CD, the lines ED, and DC, would be equal to CG. Now ED and DC are greater than EC alone, (*by the* 20. 1.) and so CG will be greater than CE: yet C being the center of the greater circle, CE and CB are equal: therefore CG will be greater than CB, which is impossible.

PROPOSITION XII.

A THEOREM.

If two circles touch each other on the outside, a line drawn through both their centers, will pass through the point where they touch.



IF the line AB, which does not pass through the point C where the circles touch, be said to be drawn from the center A to the center B; draw the lines AC and BC.

Demonstration.

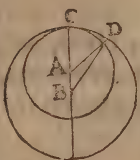
In the triangle ACB, the sides AC and BC would not be greater than the side AB alone, (which is contrary to the 20. 1.) because AD and AC, as also BE and BC, are equal.

PRO-

PROPOSITION XIII.

A THEOREM.

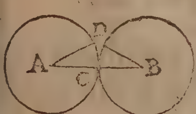
Two circles can touch each other only in one point.



FIrst, if two circles touch each other on the inside, they will touch but in one point only, the point C; which is markt out by the line BAC passing through both their centers, A, and B. For if they should touch likewise in the point D, draw the lines AD, BD.

Demonstration.

The lines AC and AD, drawn from the center of the lesser circle to its circumference, are equal: and adding AB, the lines BA, AC, and BA and AD, would be equal. Now BC and BD, drawn from the center of the greater circle to its circumference, will be equal: therefore the sides BA and AD will be equal to the side BD alone, which is contrary to the 20. 1.



Secondly if two circles touch each other on the outside; drawing the line AB from one center to the other;

it will paſs through the point C, where the circles touch, (*by the* 12.) But if you ſay that they touch alſo at the point D: having drawn the lines AD and BD; the lines BC and BD, AC and AD, being equal, the two ſides of a triangle taken together, would be equal to the third, which is contrary to the 20. 1.

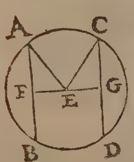
The USE.

'These four propoſitions are very clear, and 'evident; and alſo neceſſary in *Aſtronomy*, 'when we make uſe of *Epicycles*, to explain the 'motions of the *Planets*.

PROPOSITION XIV.

A THEOREM.

Equal lines drawn within a circle, are equally remote from the center; and thoſe that are equally remote from the center, are equal.



Suppoſing the lines AB and CD to be equal: I prove, that the perpendiculars EF and EG, drawn from the center, are alſo equal. Draw the lines EA and EC.

Demonſtration.

The perpendiculars EF and EG divide the lines AB and CD in the middles at the points

F

F and G, [*by the* 3.] therefore AF and CG are equal. The angles F and G are right angles: therefore [*by the* 47. 1.] the ſquare of EA is equal to the ſquares of EF and FA; as alſo the ſquare of EC is equal to the ſquares of EG and GC; but the ſquares of EA and EC are equal, becauſe the lines EA and EC are equal: therefore the ſquares of EF and FA are equal to the ſquares of EG and GC: and taking away the equal ſquares AF and CG, there will remain the ſquares of EF and EG equal; and conſequently the lines EF and EG, which are the diſtances of the lines AB and CD from the center, are equal.

But ſuppoſing the diſtances or perpendiculars EF and EG to be equal; I will prove after the ſame manner that the ſquares of EF and FA are equal to the ſquares of EG and GC; and taking away the equal ſquares of EF and EG, there will remain the ſquares of AF ind CG equal. And therefore the lines AF and CG, and their double AB and CD, are equal.

PROP.

PROPOSITION XV.

A THEOREM.

The Diameter is the greatest of all lines inscribed in a Circle; and of the rest that is the greatest which is nearest the center.

THE diameter AB is the greatest of all lines that can be drawn in the circle GIDC. As for example, it is greater than CD; for draw the lines EC and ED

Demonstration.

In the triangle CED, the sides EC and ED are greater than CD alone, [*by the* 20.1.] but AE and EB, or AB, is equal to EC and ED; therefore the diameter AB is greater than CD,

Secondly, let the line GI be more remote from the center than the line CD; that is to say, let the perpendicular EH be greater than the perpendicular EF. I say that CD is greater than GI. Draw the lines EC, and EG.

Demonstration.

The squares of CF and FE [*by the* 47.1.] are equal to the square of EC: but the square of EC is equal to the square of EG, and the square of EG

EG equal to the squares of GH and HE: therefore the squares of CF and FE are equal to squares of GH and HE; and taking from one side the square of HE, and from the other the square of EF, which is less than the square of HE, the square of CF will remain greater than the square of GH. Therefore the line CF will be greater than the line GH; and the whole line CD, the double of CF, will be greater than GI, the double of GH.

The USE.

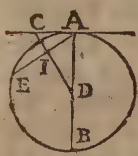
'*Theodosius* makes use of these two Propositions to demonstrate, that in a sphere the lesser circles are more remote from the center. I have also made use of them in *Astrolabes.* To these Propositions may likewise be referr'd that Mechanical proposition of *Aristotle*, by which he shows, that the Rowers at the middle of a Gally have greater force, than those that are at, either the fore, or hinder part thereof; because the sides of the Gally being crooked, the Oars of the middle part are longer, *i. e.* reach farther, than the rest. The Demonstrations relating to the *Iris*, or Rainbow, do also suppose the truth of these propositions.

THE

PROPOSITION XVI.

A THEOREM.

A line drawn perpendicularly upon the extremity of the diameter falls wholly on the outside of the circle, and touches it. But any other line drawn betwixt that and the circumference of the circle, enters within the circle, and cuts it.



LET the perpendicular AC be drawn upon the point A, which is the extremity of the diameter AB: I say first, that all the other parts of the same line, for example the point C, fall on the outside of the circle. Draw the line DC.

Demonstration.

Since the angle DAC of the triangle DAC is a right angle, DCA will be an acute: and (*by the* 19. 1.) the side DC will be greater than the side DA; therefore the line DC reaches beyond the circumference of the circle.

I add, that the line CA touches the circle, because that meeting with it at the point A it does not cut it, but all its points are on the outside of the circle.

I say also that no other line can be drawn from

from the point A below CA, which does not cut the circle. If there could, suppose EA to be such an one; and from the point D draw a perpendicular to it, DI.

Demonstration.

Since the angle DIA is a right angle, and the angle IAD an acute, AD will be greater than DI: therefore the line DI does not reach to the circumference, but the point I is within the circle.

The USE.

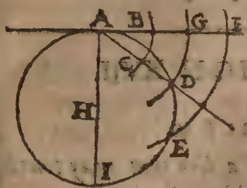
'Some Philosophers use this Proposition, but 'altogether in vain, to prove, that quantity is 'not divisible *in infinitum*, or that there really 'are in the world such things as *Zenonical*, i.e. '*absolutely and in their own nature indivisible* 'points. For the proposition does not, as they 'would have it, prove, that a circle touches a 'right line in a *Zenonical*, but in a *Mathematical point*, which is nothing else but a quanti'ty consider'd without distinction of parts, that 'is to say, without conceiving them distinct and 'separate one from the other, whether in reality 'it has such parts or not making no matter. 'We can therefore take any quantity whatsoe'ver for a *Mathematical point*; which being 'once establish'd, our Circle will consist of such 'points, and will be *mathematically* perfect, 'provided it touch not a right line, but in a 'part equal to that quantity which we have ta-

'ken for a point. But if we afterwards take a 'less part for our *Mathematical point*, the circle 'which was exactly perfect according to the 'first supposition, will be imperfect in the se-'cond, and degenerate into a Polygone. I be-'lieve, tis as impossible to describe a circle, 'that according to any supposition whatsoever 'shall be most exactly perfect, as it is to con-'ceive the least possible quantity.

'Secondly, those consequences, which some 'men draw from this proposition relating to the 'angle of contact, which they take to be less 'than any rectilineal angle, are grounded upon 'this mistake, that they imagine an angle to be 'a true quantity; the contrary of which may 'appear from hence, That the lines, that con-'tain an angle, being produc'd to any longi-'tude, the angle becomes not at all the greater. 'Further, it ought to be duly consider'd, what 'we mean, when we say, that one angle is grea-'ter than another; for this is all we under-'stand, that a circle being described from the 'point of concourse at any distance whatsoever, 'the lines of that we call the greater angle will 'contain betwixt them a greater arch of that 'circle, than those of that which we call the 'less; which is the sole meaning of the Excess 'of one angle above another. From whence I 'infer, that the angle of contact can no more be 'compar'd with a rectilineal angle, than a su-

perficies

'superficies with a line, being at the same time 'both equal, and greater and less than a recti-lineal angle. As for ex-'ample; from the point 'A draw the line AD, 'making with AE a re-'ctilineal angle; I say 'it is both greater and 'less than, and equal to, 'the angle of contact.



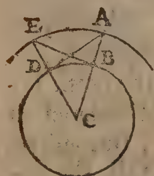
'For if we suppose divers circles describ'd 'from the point A, as the center, whereby to 'measure those angles: it is evident that, ac-'cording to the arch drawn beyond the point D, 'that is the arch EF, the angle of contact is 'greater than the rectilineal angle. But on the 'contrary, according to the arch CB, the re-'ctilineal angle is the greater of the two. And lastly, according to the arch DG, passing through the point in which AD cuts the circumference, they are both equal. From whence it follows, that the angle of contact is at the same time both less and greater than, and equal to, the rectilineal angle: and consequently, they ought not at all to be compar'd together. In a word, Angles are no quantities; nor are they call'd less or greater one than another, but with respect to the arches which they contain: so that all the disputes about the angle of contact, and all the Para-

'doxes, conclude nothing either for or against
'the divisibility of quantity; an Angle being
'no species, but only a property thereof.

PROPOSITION XVII.

A PROBLEM.

From a point given to draw a line that may touch a Circle.



TO draw a line from the point A touching the circle BD, draw the line AC to its center; and at the point B draw a perpendicular BE, which may cut an arch of a circle, describ'd from the center C through the point A, at the point E. Draw also the lines EC, and AD. I say the line AD touches the circle in D.

Demonstration.

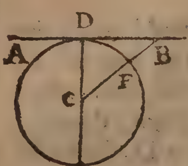
The triangles EBC and ADC have the same angle C; and the sides CD and CB, CE and CA, equal, (*by the definit. of a Circle:*) and therefore they are equal in all respects, (*by the* 4. 1.) and the angles CBE and CDA are equal. But the angle CBE is a right angle, therefore the angle CDA will be so too, and (*by the* 16.) the line AD will touch the circle.

PRO-

PROPOSITION XVIII.

A THEOREM.

A line drawn from the center of a circle to the point where a right line touches it, is perpendicular to that line.



IF the line CD be drawn from the center C to the point of contact D, CD will be perpendicular to AB. For if it be not, draw the line CB perpendicular to AB.

Demonstration.

Since the line CB is suppos'd to be perpendicular, the angle B will be a right angle, and consequently CDB an acute, (*by the* 32. 1.) Therefore the line CB, oppos'd to the lesser angle, will be less than CD, which is impossible; because CF, which is but part of CB, is equal to CD.

PROP.

PROPOSITION XIX.

A Theorem

If a line, perpendicular to the tangent, be drawn from the point of contact, it will pass through the center of the circle.

LET the line AB [*see Fig. preced.*] touch the circle at the point D, and the line DC be perpendicular to AB. I say, that DC passes through the center. For if it did not, drawing a line from the center to the point D, it would be perpendicular to AB, (*by the preceding*) and so there would be two perpendiculars drawn to the same point D of the same line, which cannot be.

The Use.

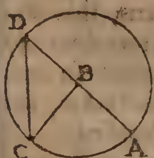
'The use of lines *Tangents* is very common 'in *Trigonometry*; upon which account it is that 'I have made a table, whereby to measure all sorts 'of triangles, as well spherical as rectilineal. In 'my *Opticks* likewise are divers propositions 'founded upon *Tangents*; as when is determin'd 'what part of a Globe is enlighten'd. The *pha-* '*ses* or Apparitions of the Moon are establish'd 'also upon the same doctrine; and that famous 'Problem of *Hipparchus's*, by which he found 'the distance of the Sun, by the difference of the

'the true and apparent *Quadratures.* In Dialling the *Italian* and *Babylonian* hours are frequently describ'd by lines Tangents. Lastly, 'we take the dimensions of the earth by a line 'that touches its superficies; and in the art of 'Navigation, take a Tangent line for our Horizon.

PROPOSITION XX.

A THEOREM.

The angle at the center is double the angle at the circumference, which has the same arch for its base.



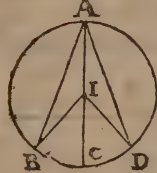
IF the angle ABC, which is at the center, and the angle ADC, at the circumference, have the same arch AC for their base, the first will be double the second. This proposition has three different cases: the first of which is, when the line ABD passes through the center B, the line AB in one triangle concurring with the line BD of the other.

Demonstration.

The angle ABC is the external angle in respect of the triangle BDC: therefore (*by the* 32. 1.) it is equal to both the angles D and C: which

which being equal, *(by the* 5. 1.) becauſe their ſides BC and BD are equal, the angle ABC is the double of either.

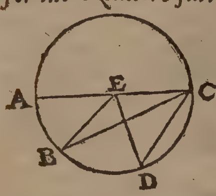
The ſecond caſe is, when one angle incloſes the other, but none of the lines that form them concur in one; as you ſee in the next figure. The angle BID is at the center, and the angle BAD at the circumference. Draw the line AIC through the center.



Demonſtration.

The angle BIC is double the angle BAC, and CID is double the angle CAD, *(by the preceding caſe :)* therefore the angle BID is double the angle BAD.

The third caſe is, when it happens, that neither one angle incloſes the other, nor does any of the lines that form them, concur in one. *Which caſe is wholly omitted by my Author, but for the Readers ſatisfaction is here ſupplied.*



Let the angle at the center be BED, and the angle at the circumference BCD, having the ſame arch for their baſe BD. I ſay, the angle BED is double the angle BCD. Draw the line EC, and continue it to the point A.

Demon-

Demonstration.

"The angle AED is double the angle ACD, (*by the* 1. *case*;) and (*by the same*) the angle AEB is double the angle ACB: therefore the remainder of the one BED is double the remainder of the other BCD.

The USE.

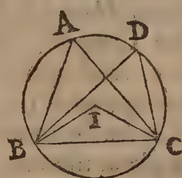
' That Problem, which is ordinarily pro-
' pos'd, showing how to describe an *Horizontal*
' *Dial* by one sole opening of the Compass, is
' built in part on this Proposition. And again,
' when we would determine the *Apogæum* of the
' Sun, or the excentricity of his Circle, by three
' observations, we suppose the angle at the cen-
' ter to be double that at the circumference.
' *Ptolomey* makes frequent use of this proposition
' to determine both the excentrick circle of the
' Sun, and the Epicycle of the Moon. The first
' Proposition of the third book of *Trigonome-*
' *try* is grounded also upon this here.

X PROP.

PROPOSITION XXI.

A THEOREM.

The angles, that are in the same segment of a circle, or that have the same arch for their base, are equal.



IF the angles BAC and BDC are in the same segment of a circle, which is greater than a semicircle, they will be equal. Draw the lines BI and CI.

Demonstration.

The angles A and D are each of them the half of the angle BIC, (*by the preceding*,) therefore they are equal. They have likewise the same arch BC for their base.

Secondly, let the angles A and D be in the same segment BAD, which is less than a semicircle; they will nevertheless be equal.

Demonstration.

All the angles of the triangle ABE are equal to all the angles of the triangle DEC, (*by* 1.*Coroll. of the* 32.1.) but the angles AEB and DEC are equal, (*by the* 15. 1.) Also the angles ECD and

and ABE are equal, (*by the preceding caſe*,) being in the ſame ſegment ABCD, greater than a ſemicircle; therefore the angles BAE and EDC are equal; which, the angles at E being equal, and conſequently (*by the Coroll. of the* 15. 1.) the lines AE and EC, making but one right line, as likewiſe DE and EB another, are the angles A and D, in the ſame ſegment ABCD, and having the arch BC for their baſe.

The USE.

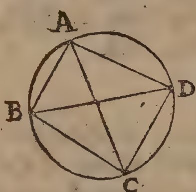
'This Propoſition is produc'd in *Opticks* to 'prove, that the line BC will appear of the 'ſame greatneſs, when tis view'd from A, and 'D, becauſe it is ſeen in both caſes under e-'qual angles.

'The ſame propoſition is us'd to deſcribe large 'circles without having their centers; for ex-'ample, if we would make large Copper baſons of 'a ſpherical figure, ſuch as we might work upon 'in poliſhing Spectacles, and glaſſes to ſee at a 'great diſtance. For having made in Iron an an-'gle BAC equal to that, which is contain'd in 'the ſegment ABC, and at the points B and C 'ſtrongly faſten'd two ſmall iron pins; if the 'triangle BAC be mov'd ſo, that the ſide AB 'may always touch the pin B, and the ſide AC 'the pin C, the point A will deſcribe an arch 'of the circle ABCD. This manner of deſcrib-'ing a circle may alſo be us'd in making great 'Aſtrolabes.

PROPOSITION XXII.

A Theorem.

Quadrilateral figures, inscrib'd in a circle, have their opposite angles equal to two right angles.



LET a quadrilateral figure, or a figure of four sides, be inscrib'd in a circle, in such sort that all its angles may terminate at the circumference of the circle ABCD: I say the opposite angles BAD and BCD are equal to two right angles. Draw the diagonals AC, and BD.

Demonstration.

x 32,1

All the angles of the triangle BAD are equal to two right angles, In stead then of the angle ABD put the angle ACD, which is equal to it, (*by the* 21.) being in the same segment ABCD: and instead of the angle ADB, put the angle ACB, which is in the same segment of a circle BCDA. Therefore the angles BAD, and the angles ACD and ACB, that is to say, the whole angle BCD, are equal to two right angles.

The USE.

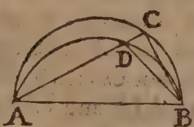
'*Ptolomey* makes use of this Proposition to 'frame the table of Chords, or lines subtend-ing

'ing arches. I have alſo us'd the ſame in my 'third book of T*rigonometry*, to prove, that the 'ſides of an obtuſangle triangle would have the 'ſame proportion among themſelves as the 'ſines of the oppoſite angles.

PROPOSITION XXIII.

A THEOREM.

Two ſimilar ſegments of a circle, deſcrib'd upon the ſame line, are equal.



I call thoſe ſimilar ſegments of a circle, which contain equal angles; and I ſay, that if ſuch be deſcrib'd upon the ſame line AB, they will fall one upon the other, and not exceed each other in any part. For if either did exceed the other, as do the ſegments ADB, and ACB, they would not be ſimilar: to demonſtrate which, draw the lines ADC, BD and BC.

Demonſtration.

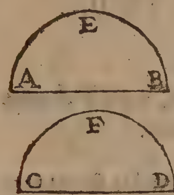
The angle ADB is an external angle in reſpect of the triangle DBC: therefore (*by the* 16. 1.) it is greater than the angle ACB, and by conſequence the ſegments ADB and ACB contain unequal angles, which I ſay is to be diſſimilar.

PROP.

PROPOSITION XXIV.

A THEOREM.

Two similar segments of a circle describ'd upon equal lines, are equal.



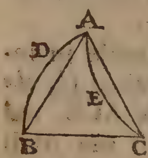
IF the segments of the circles AEB, and CFD, be similar, and the lines AB and CD equal, the segments also will be equal.

Demonstration.

Suppose the line CD to be plac'd upon the line AB, being suppos'd to be equal, they will not exceed each other; and then the segments AEB and CFD will be describ'd upon the same line, and therefore will be equal, (*by the preceding.*)

The USE.

'Crooked figures are frequently reduc'd to 'rectilineals by this proposition. As for example: if two similar segments 'of a circle AEC, and ADB, be 'describ'd upon AB and AC, the 'the equal sides of the triangle 'ABC: tis evident, that, transposing the segment AEC unto ADB, the triangle ABC is equal to the figure ADBCEA.

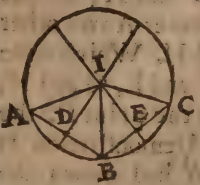


PROP.

PROPOSITION XXV.

A PROBLEM.

To compleat a circle, of which we have but a part.



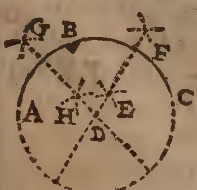
HAving the arch ABC given; to compleat the circle we must find its center; to which end draw the lines AB and BC, which having divided in the middles at the points E and D, draw their two perpendiculars EI and DI; which will meet at the point I, the center of the circle.

Demonstration.

The center is in the line DI, (*by the coroll. of the* 1.) it is also in EI, (*by the same;*) therefore it must be at the point I.

The USE.

'This Proposition occurs very frequently; 'but sometimes it is express'd in other terms: 'as to inscribe a triangle in a circle; or to de- 'scribe a circle through three 'points given, provided they 'be not plac'd in a right line.



'Let the points propos'd be 'A, B, and C; and placing 'the foot of the compass at 'the point C, describe two arches

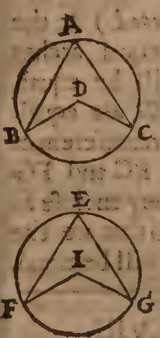
'arches F and E, at any diſtance whatſoever. 'Then remove the foot of the compaſs to the 'point B, and at the ſame diſtance deſcribe two 'other arches cutting the former in E and F; 'alſo from the point B, as the center, deſcribe 'at any diſtance the arches G and H, and at 'the ſame diſtance from the center A two other 'arches cutting them in G and H. Which done 'draw the lines through F and E, G and H, 'which ſhall cut each other at the point D, the 'center of the circle. The Demonſtration is 'obvious enough; for if you had drawn the 'lines AB, and BC, you had, by this opera-'tion, divided them equally and perpendicu-'larly. This propoſition is exceeding neceſſa-'ry to deſcribe Aſtrolabes, and compleat cir-'cles, of which we have but three points. That 'Propoſition in *Aſtronomy*, which teaches how 'to find the *Apogæum*, and excentricity of the 'circle of the Sun, virtually contains this. And 'I alſo have made frequent uſe of it in my 'Treatiſe concerning the Cutting of Stones.

PROP.

PROPOSITION XXVI.

A Theorem.

Equal angles, whether at the centers, or the circumferences of equal circles, have equal arches for their bases.



IF the angles D and I, at the centers of equal circles ABC, and EFG, be equal; the arches BC and FG will be equal. For if the arch BC was greater or less than the arch FG, since the angles are measur'd by arches, the angle D would be greater or less than the angle I.

But if the equal angles be suppos'd to be at the circumferences of equal circles, as A and E; the angles which they enclose at the centers, as D and I, being their doubles, will be likewise equal, and consequently require equal arches for their bases, BC and FG; which arches are likewise the measures of the angles A and E.

§20

PROPOSITION XXVII.

A Theorem.

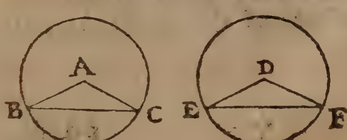
Angles, whether at the centers or circumferences of equal circles, having equal arches for their bases, are also equal.

If the angles D and I (*fig. praced.*) at the centers of equal circles have equal arches BC and FG for their bases, they will be equal, because their measures BC and FG are equal. And if the angles A and E, at the circumferences of equal circles have equal arches BC and FG for their bases, since the angles they enclose at the centers will be equal, they also that are the halves of those angles (*by the* 20.) will be equal.

PROPOSITION XXVIII.

A Theorem.

Equal lines, within equal circles, answer to equal arches.



If the equal lines BC and EF be applied to equal circles, ABC, and

and DEF, they will be the chords of equal arches, BC, and EF. Draw the lines AB, AC, ED, EF.

Demonstration.

In the triangles ABC and DEF, the ſides AB and AC, DE and DF are equal, being the ſemidiameters of equal circles: and their baſe's BC and EF are ſuppos'd equal, therefore (*by the* 8.1.) the angles A and D will be equal; and (*by the* 26.) the arches BC and EF will be alſo equal.

PROPOSITION XXIX.

A THEOREM.

The lines that ſubtend equal arches of equal circles are equal.

IF the lines BC and EF (*ſee fig. preced. Prop.*) ſubtend (or are the chords of) equal arches BC and EF in equal circles, they will be equal.

Demonſtration.

The arches BC and EF are equal, and parts of equal circles; therefore (*by the* 27.) the angles A and D will be equal. Therefore in the triangles ABC, DEF, the ſides AB, AC, DE, and DF being equal, as alſo the angles A and D; the baſes BC, EF will be equal, (*by the* 4.1.)

The USE.

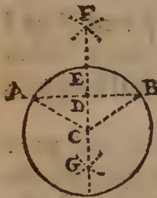
Theodoſius by the 28 and 29 demonſtrates,

'that the arches of the circles of the *Italian* 'and *Babylonian* hours, contain'd between two 'parallels, are equal. We have alſo demon-'ſtrated after the ſame manner, that the arches 'of the circles of the Aſtronomical hours, con-'tain'd between two lines parallel to the *Equa-'tor*, are likewiſe equal. Theſe Propoſitions are 'almoſt of continual uſe in ſpherical *Trigono-'metry*, and alſo in *Dialling*.

PROPOSITION XXX.

A PROBLEM.

To divide an arch of a circle into two equal parts.



SUppoſe the arch AEB was to be divided into two equal parts, Place the foot of the compaſs at the point A, and deſcribe two arches F and G; then removing it to the point B at the ſame diſtance deſcribe other two arches, cutting the former in F and G; the line GF will cut the arch AB equally at the point E. Draw the line AB.

Demonſtration.

By this operation you have divided the line AB into two equal parts. For ſuppoſe there were drawn

drawn the lines AF, BF; AG, and BG; (which I have not done, least the figure should appear confus'd,) the triangles FGA and FGB would have all their sides equal, therefore (*by the* 8.1.) the angles AFD,& BFD would be equal. Again, the triangles DFA and DFB have the side DF common, the sides AF and BF equal, and the angles DFA and DFB equal: therefore (*by the* 4. 1.) the bases AD and BD are equal, and also the angles ADF and BDF. We have therefore divided the line AB equally and perpendicularly at the point D. Therefore (*by the* 1.) the center of the circle is in the line FG. Suppose it then to be the point C, and draw the lines CA and CB; all the sides of the triangles ACD and BCD are equal: therefore [*by the* 8. 1.] the angles ACD and BCD are equal, and [*by the* 26.] the arches AE and EB.

The USE.

'Having frequent occasion to divide an arch 'into two equal parts, the exercise of this Pro'position is very common. Tis thus that we 'divide the Mariners compass into 32 winds: 'for having drawn two diameters cutting each 'other at right angles, we divide the circle in'to four, and subdividing each quarter in the 'middle, we have eight, parts; and again sub'dividing those twice, we make 32. We have

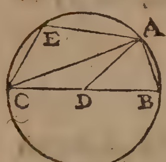
also

‘ alſo occaſion for the ſame operation in the di-
‘ viding a ſemicircle into 180 degrees; and
‘ becauſe to compleat that diviſion we are ob-
‘ lig'd to divide an arch into three equal parts,
‘ all *Geometricians* have ſought after a method
‘ of doing that *Geometrically*, but have not yet
‘ been ſo happy as to find one.

PROPOSITION XXXI.

A Theorem.

The angle in a ſemicircle is a right angle, that which is in a ſegment greater than a ſemicircle is an acute, and that which is in a leſſer ſegment is an obtuſe.



If the angle BAC be in a ſemicircle, I will prove that it is a right angle. Draw the line DA. *Demonſtration.*

The angle ADB being an external angle in regard of the triangle DAC, is equal to both the internals DAC and DCA (*by the* 32.1.) and thoſe being equal (*by the* 5.1.) becauſe the ſides DA and DC are equal, it will be double the angle DAC. In like manner the angle ADC is double the angle DAB: therefore the two angles ADB, and ADC, which are equal to two right angles, are double the

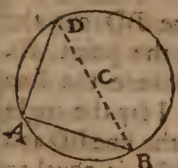
the whole angle BAC, and consequently the angle BAC is a right angle.

Secondly, the angle AEC, which is in the segment AEC less than a semicircle, is an obtuse angle. For in the quadrilateral figure AB CE, the two opposite angles E and B are equal to two right angles, (*by the* 22.) but the angle B is an acute; therefore the angle E will be an obtuse.

Thirdly, the angle B, which is in the segment ABC greater than a semicircle, is an acute; because in the triangle ABC, the angle BAC is a right angle.

The USE.

'*Mechanicks* make use of this Propositi-'on to try if their Squares 'be just; for having describ'd 'a semicircle BAD, they lay 'down the point A of their 'square BAD upon the cir-'cumference, and one of its 'sides AB upon the point of the 'diameter B: and then the other branch AD 'ought to pass precisely to the point D, which 'is the other extreme of the diameter.



'*Ptolomey* uses this Proposition to compose 'his table of Chords or Subtendants, of which 'he has occasion in his *Trigonometry.*

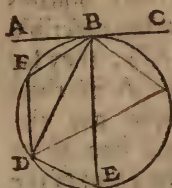
'There is also a method of raising a perpen-dicular

'dicular at the end of a line, grounded upon 'this proposition. For example, to raise a perpen-'dicular at the point A of the line AB. I place 'the foot of the compass upon the point C ta-'ken any where, and describe a circle through 'the point A, cutting the line AB at the point 'B. Then I draw the line BCD; and so 'tis 'evident, that the line AD is in a semicircle.

PROPOSITION XXXII.

A THEOREM.

A line cutting a circle at the point of contact, makes, with the tangent, angles, equal to those in the alternate segments.



LET the line BD cut the circle at the point B, which is that where the line AB touches it. I say the angle CBD, made by the line BD and the tangent ABC, is equal to the angle F in the alternate segment BFD; and that the angle ABD is equal to the angle E in the segment BED.

First, if the line pass through the center, as the line BE, it will make with the tangent two right angles, (*by the* 18.) and the angles of the semicircles would be also right angles, (*by the* pre-

preceding,) therefore in this caſe the propoſition would be true. But if the line do not paſs through the center, as BD; draw the line BE through the center, and joyn the line DE.

Demonſtration.

The line BE makes with the tangent two right angles; and all the angles of the triangle BDE are equal to two right angles, (*by the* 32. 1.) therefore taking away the right angles CBE, and D which is in the ſemicircle, and likewiſe the angle EBD which is common to both, there will remain the angle ABD equal to the angle E.

Again, the angle CBD is equal to the angle F; becauſe in the quadrilateral figure BFDE, which is inſcrib'd in a circle, the oppoſite angles E and F are equal to two right angles, [*by the* 22.] but the angles ABD and CBD are alſo equal to two right angles, [*by the* 13. 1.] and the angles ABD and E are equal, as I have now demonſtrated: therefore the angles CBD and F are equal.

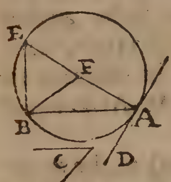
The USE.

'This propoſition is neceſſary to prove that 'which follows.

PROPOSITION XXXIII.

A PROBLEM.

Upon a line given to describe a segment of a circle capable of an angle given.



LET it be propos'd to describe a segment of a circle upon the line AB capable of the angle C. Make the angle BAD equal to the angle C, and draw AE perpendicular to AD; make also the angle ABF equal to the angle BAE: and in fine, from the point F, where BF and AE concur, at the distance BF or FA, describe a circle. The segment BEA is capable of an angle equal to the angle C.

Demonstration,

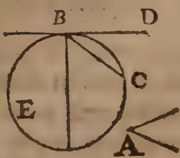
The angles BAF and ABF being equal, the lines FA and FB are equal, [*by the* 6. 1.] and the circle, which is describ'd from the center F, by A, passes by B: Now the angle DAE being a right angle, the line DA touches the circle in A, [*by the* 16.] therefore the angle contain'd in the segment BEA, as the angle E, is equal to the angle DAB, that is the angle C, [*by the preceding.*] But if the angle given be an obtuse, we must take an acute, its complement to 180 degrees.

PROP.

PROPOSITION XXXIV.

A PROBLEM.

A circle being given, to cut a ſegment in it capable of a certain angle.



TO cut a ſegment of the circle BCE capable of the angle A, draw [*by the* 17.] the tangent BD, and make the angle DBC equal to the angle A. Tis evident [*by the* 32.] that the ſegment BEC is capable of an angle equal to DBC, and conſequently to the angle A.

The USE.

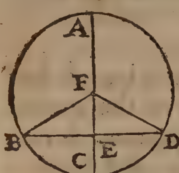
'I have made uſe of this propoſition to find '*Geometrically* the excentricity of the Annual 'circle of the Sun, and his *Apogæum*, having 'three obſervations given. Tis uſed likewiſe 'in *Opticks*, to find a point where two unequal 'lines propos'd may appear equal, or under 'equal angles, by making upon each line ſeg- 'ments which will contain equal angles.

PROPOSITION XXXV.

A THEOREM

If two lines cut each other within a circle, the rectangle contain'd under the parts of one is equal to the rectangle contain'd under the parts of the other.

FIrst, if the two lines cut each other in the center, they will be both equal, and both equally divided; so that in that case it is evident, the rectangle contain'd under the parts of one, will be equal to the rectangle contain'd under the parts of the other.



Secondly, if one of the lines pass through the center F, as AC, and divide the line BD, into two equal parts at the point E: I say, the rectangle contain'd under AE and EC is equal to the rectangle contain'd under BE and ED, that is to say, to the square of BE. The line AC is perpendicular to BD, [*by the* 3.]

Demonstration.

Since the line AC is divided equally in F, and unequally in E, the rectangle contain'd under AE, and EC, with the square of FE, is equal

to

to the ſquare of FC or FB, [*by the* 5. 2.] Now the angle E being a right angle, the ſquare of FB is equal to the ſquares of BE and FE; there- £47,1. fore the rectangle under AE, EC, with the ſquare of FE, is equal to the ſquares of BE and EF: and taking away the ſquare of EF, there remains the rectangle under AE, EC, equal to the ſquare of BE.

Thirdly, let the line paſs through the center F, and divide the line CD into unequal parts at the point E: draw FG perpendicular to CD, and [*by the* 3.] the lines CG and GD will be equal.

Demonſtration.

Since the line AB is divided equally in F and unequally in E, the rectangle contain'd under AE, EB, with the ſquare of EF, is equal to the ſquare of FB, or FC, [*by the* 5. 2.] Inſtead of the ſquare of EF put the ſquares of FG and GE, which are equal to it, [*by the* 47. 1.]

In like manner the line CD being divided equally in G, and unequally in E; the rectangle under CE, ED, with the ſquare of GE, is equal to the ſquare of GC. Add the ſquare of GF; the rectangle under CE, ED, with the ſquares of GE and GF, will be equal to the ſquares of CG and GF, that is to ſay, [*by the* 47. 1.] to the ſquare of FC. Therefore the rectangle under

der AE, EB, with the squares of EG and GF, is equal to the rectangle under CE, ED, with the same squares: and consequently taking away the same squares from both, the rectangle AE, EB, will be equall to the rectangle CE, ED.

Fourthly, if the lines CD and HI, cut each other in E, neither of the two passing through the center: I say, the rectangle CE, ED is equal to the rectangle HE, EI. For drawing the line AFB, it is plain the rectangles CE, ED, and HE, EI, are both equal to the rectangle AE, EB, [*by the preceding case*;] therefore they are equal betwixt themselves.

The USE.

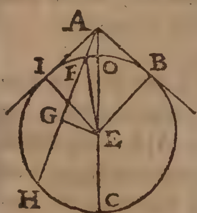
We are taught by this Proposition a method 'of finding a fourth proportional to three lines 'given, or a third proportional to two,

PROP.

PROPOSITION XXXVI.

A THEOREM.

If from a point taken without the circle a line be drawn to touch, and another to cut, the circle: the square of the Tangent will be equal to the rectangle contain'd under the whole secant, and the external line.



Suppose the line AB to be drawn from the point A, taken without the circle, to touch the circle in B; and the line AC, or AH cutting it. The square of AB will be equal to the rectangle contain'd under AC, and AO, as also the rectangle contain'd under AH, and AF. If the secant pass through the center, as AC, draw the line EB.

Demonstration.

Since the line OC is divided in the middle at the point E, and the line AO added to it; the rectangle contain'd under AO and AC, with the square of OE or EB, will be equal to the square of AE, [*by the 6.2.*] Now the line AB is suppos'd to touch the circle at the point B: therefore [*by the* 18.] the angle B is a right angle,

gle, and (*by the* 47. 1.) the ſquare of AE is e-qual to the ſquares of EB and AB; therefore the rectangle under AC and AO, with the ſquare of EB, is equal to the ſquares of EB and AB: and taking away the ſquare of EB from both, the rectangle under AC, AO will be e-qual to the ſquare of AB.

Secondly, ſuppoſe the ſecant AH not to paſs through the center; and draw the line EG perpendicular to EH, which will divide in the middle the line FH at the point G; draw alſo the line EF.

Demonſtration.

The line FH being divided equally at the point G, and the line AF being added to it; the rectangle contain'd under AH, AF, with the ſquare of FG, will be equal to the ſquare of AG. Add to both the ſquare of EG: the rectangle under AH, AF, with the ſquares of FG and GE, that is (*by the* 47.1.) the ſquare of FE, or EB, will be equal to the ſquares of AG and GE, that is, (*by the* 47.1.) the ſquare of AE. Further, the ſquare of AE (*by the ſame*) is equal to the ſquares of EB and AB: therefore the rectangle contain'd under AH, AF, with the ſquare of BE, is equal to the ſquares BE and AB: and taking away the ſquare of BE from both, the rectangle contain'd under AH, AF will be equal to the ſquare of AB.

Coroll. 1. If you draw divers ſecants from the ſame

ſame point, as AC and AH, the rectangles under AC and AO, AH and AF, will be equal betwixt themſelves, ſince they are both equal to the ſquare of AB.

Coroll. 2. If you draw two tangents from the ſame point, as AB, AI, they will be equal: becauſe the ſquares will be equal to the ſame rectangle under AC, and AO; and conſequently betwixt themſelves; as alſo the lines.

PROPOSITION XXXVII.

A THEOREM.

If the rectangle contain'd under the ſecant and the external line be equal to the ſquare of a line that falls upon the circle, that line will touch the circle.

SUppoſe the ſecant to be AC or AH, and the rectangle AC, AO; or AH, AF, (*ſee fig. preced.*) to be equal to the ſquare of the line AB; the line AB will touch the circle. Draw the tangent AI, (*by the* 17.) and the line IE.

Demonſtration.

Since the line AI touches the circle, the rectangle AC, AO; or AH, *AF*, will be equal to the ſquare of AI. But the ſquare of AB is ſuppos'd to be equal to either of thoſe rectangles; therefore the ſquares of AI and AB are equal, and conſequently the lines AI and AB. Therefore the

the triangles ABE and AIE, having all ſides equal, will be equiangular, (*by the* 8. 1.) and because the angle AIE is a right angle (*by the* 18.) the line AI being a tangent, the angle ABE will be a right angle, and the line AB a tangent, (*by the* 16.)

The USE.

‘*Maurylocus* makes uſe of this Propoſition to ‘to find the diameter of the Earth. For obſerv‘ing from the top of a mountain OA, the ſu‘perficies of the earth by the line BA, he ‘takes notice of the angle OAB, made by the ‘line AB and a perpendicular AC: and by *Tri*‘*gonometry* calculates the length of the line AB. ‘Then multiplying AB by AB to have its ‘ſquare, he divides the product by AO the ‘height of the mountain, which gives the quo‘tient AC, the diameter of the earth, with the ‘height of the mountain; from which having ‘ſubducted AO, there will remain OC the ‘diameter of the earth. This propoſition ſerves ‘alſo to prove the fifth of the third book of ‘*Trigonometry*.

THE

THE FOURTH BOOK OF THE ELEMENTS OF EUCLID.

'THis fourth book is exceeding uſeful in
'*Trigonometry*. For by inſcribing Poly-
'gons in a Circle, we learn the methods of
'compoſing the Table of Subtendants, Tan-
'gents, and Secants; a practice moſt neceſſary
'for taking all ſorts of Dimenſions.

'Again, by inſcribing Polygons in a circle, we
'find the divers Aſpects of the Stars, which al-
'ſo take their names from thoſe Polygons.

'Thirdly, the ſame operations give us the
'*Quadrature* of the *Circle*, as exact as is needful.
'And by them we alſo demonſtrate, that Cir-
'cles are in the duplicate proportion of that of
'their Diameters.

'Fourthly, Military Architecture does fre-

quently

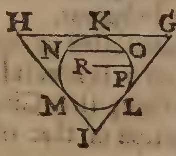
'quently require to inſcribe Polygons in a cir-'cle, to compoſe the deſigns and platforms of 'regular Fortifications.

DEFINITIONS.

1. A Rectilineal figure is inſcrib'd in a circle, or a circle is deſcrib'd about it, when all its angles are in the circumference of the ſame circle.

'As the triangle ABC is inſcrib'd in a circle, and the circle is deſcrib'd 'about the triangle; becauſe its angles A, B, 'and C, do all terminate at the circumference. 'The triangle DE*F* is not inſcrib'd in the cir-'cle, becauſe the angle D does not terminate at 'the circumference of the circle.

2. A rectilineal figure is deſcrib'd about a circle, and the circle inſcrib'd within that figure, when all the ſides of the figure touch the circumference of the circle. " as the trian-'gle GHI is deſcrib'd about 'the circle KLM becauſe its 'ſides touch the circumference 'of the circle in K, L, and M.



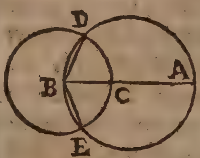
3. A line is apply'd to, or inſcrib'd in a cir-cle,

cle, when its two extreams touch the circumference of the circle. " As the line NO. But " the line RP is not inscrib'd in the circle.

PROPOSITION I.

A PROBLEM.

To inscribe in a circle a line, that does not exceed its diameter.



LEt a line be propos'd to be inscrib'd in the circle AEBD, not exceeding its diameter. Take the length of the line propos'd upon the diameter; for example, let it be BC. Place the foot of the compass upon the point B, and describe a circle at the distance of BC, which may cut the circle AEBD in D and E. Then draw the line BD or BE. Tis evident they are equal to BC, (*by the definition of a circle.*)

The USE.

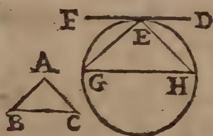
" This proposition is necessary for the performance of what is requir'd in the following.

PROP.

PROPOSITION II.

A PROBLEM.

To inscribe in a circle a triangle equiangular to another triangle.



LET the circle be EGH, in which a triangle is to be inscrib'd, equiangular to the triangle ABC. Draw the tangent *FED*, (*by the* 17. 3.) and at the point of contact E make the angle DEH equal to the angle B, and the angle *FEG* equal to the angle C, [*by the* 23. 1.] and draw the line GH; the triangle EGH will be equiangular to the triangle ABC.

Demonstration.

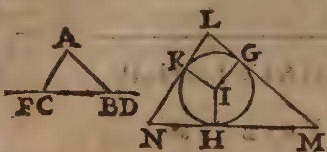
The angle DEH is equal to the angle EGH of the alternate segment, [*by the* 32. 3.] But the angle DEH is equal to the angle B, and consequently the angles B and G are equal. By the same reason the angles C and H are also equal, and [*by Coroll.* 2. *of the* 32. 1.] the angles A and GEH will be equal. Therefore the triangles EGH and ABC are equiangular.

PROP.

PROPOSITION III.

A PROBLEM.

To describe a triangle about a circle equiangular to another triangle.



IF you would describe a triangle equiangular to the triangle ABC about the circle GKH. Continue one of the sides of the triangle given BC to D and F, and make the angle GIH equal to the angle ABD, and HIK equal to the angle ACF: then draw the tangents LGM, LKN, and NHM, through the points G, K, and H. These tangents will concur; because the angles IKL and IGL being right angles, if you should draw a line KG, the angles KGL and GKL would be less than two right angles: therefore the lines GL and KL must concur, [*by the* 11. *Axiom.*]

Demonstration.

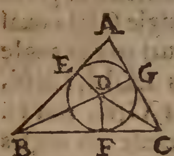
All the angles of the quadrilateral GIHM are equal to four right angles, because it may be divided into two triangles. The angles IGM and IHM, which are made by the tangents, are

£ 10, 3 are right angles; therefore the angles M and I are equal to two right angles, as are alſo the angles ABC and ABD. But the angle GIH is equal to the angle ABD, therefore the angle M will be equal to the angle ABC. By the ſame reaſon the angles N and ACB are equal, and therefore the triangles LMN and ABC are equiangular.

PROPOSITION IV.

A PROBLEM.

To inſcribe a circle in a triangle.



IF you would inſcribe a circle in the triangle ABC, divide the angles ABC and ACB into two equal parts, [*by the* 9. 1.] drawing the lines BD and CD, which will concur at the point D. This done, from the point D draw the perpendiculars DE, DF, and DG, which will be equal; ſo that a circle deſcrib'd from the center D, at the diſtance DE, will paſs through F and G.

Demonſtration.

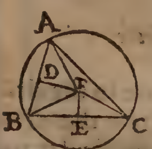
The triangles DEB and DFB have the angles DEB and DFB equal, being both right angles: the angles DBE and DBF are alſo equal, the angle

angle ABC having been divided into two equal parts; and the side DB is common: therefore (*by the* 26.1.) the triangles will be equal in all respects, and the sides DE and DF will be equal. After the same manner might I demonstrate the sides DF and DG to be equal. Tis possible therefore to describe a circle, which shall pass through the points E, F, and G; and because the angles E, F, and G are right angles, the sides AB, AC, and BC will touch the circle, which by consequence is inscrib'd in the triangle. 163

PROPOSITION V.

A PROBLEM.

To describe a circle about a triangle.



IF you would describe a circle about the triangle ABC, divide the sides AB and BC into two equal parts, at the points D and E, drawing the perpendiculars DF and EF, which will concur at the point *F*. Which done, if you describe a circle from the center *F*, at the distance *FB*, it will pass through A and C; that is to say, the lines *FA*, *FB*, and *FC*, are equal.

Demonstration.

The triangles ADF and BDF have the side

DF common, and the sides AD and DB equal, the side AB having been divided equally in D; and the angles at D are equal, being right angles. Therefore (*by the* 4. 1.) the bases *AF* and *BF* are equal; as also the bases *BF* and *CF*.

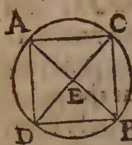
The USE.

'I have frequent occasion to inscribe a 'triangle in a circle; as, for instance, in the 'first proposition of my 3. book of Trigonome-'try. This performance also is necessary for 'the measuring the *area* of a triangle; and up-'on many other occasions.

PROPOSITION VI.

A PROBLEM.

To inscribe a square in a circle.



TO inscribe a square in the circle ACBD, draw the diameter AB, and perpendicular to it the line DC passing through the center E; then draw the lines AC, CB, BD, DA, and you will have inscrib'd in the circle the square AC BD.

Demonstration.

The triangles AEC and CEB have their sides equal, and the angles AEC and CEB equal, being

being both right angles : therefore their bases AC and CB are equal, (*by the* 4. 1.) Further, because the sides AE and EC are equal, the angles EAC and ECA will be equal; and the angle E being a right angle, they will be half-right angles, (*by the* 32.1.) therefore the angle ECB is half a right angle, and consequently the angle ACB will be a right angle. And the same reason holds for all the rest: therefore the figure ACBD is a square.

PROPOSITION VII.

A PROBLEM.

To describe a square about a circle.

HAving drawn the two diameters AB, and CD, which cut each other perpendicularly at the center E, draw the tangents FG, GH, HI, and IF, by the points A, D, B, C, and you will have describ'd the square FGHI, about the circle ACBD.

Demonstration.

The angles E and A are right angles, therefore (*by the* 27.1.) the lines FG and CD are parallels. After the same manner I may prove, that CD and HI, FI and AB, AB and GH, are parallels. Therefore the figure FCDG is a

parallelogram, and (*by the* 34. 1.) the lines *FG* and CD are equal, as alſo the lines CD and IH, *FI* and AB, AB and GH; and conſequently the ſides of the figure *FG* and HI are equal. Further, ſince the lines FG and CD are parallels, and the angle CDG is a right angle, the angle G will alſo be a right angle, (*by the* 29.1.) After the ſame manner I may demonſtrate the angles *F*, H, and I, to be right angles. Therefore the figure FGHI is a ſquare, whoſe ſides touch the circle.

PROPOSITION VIII.

A PROBLEM.

To inſcribe a circle in a ſquare.

IF you would inſcribe a circle in the ſquare *F*GHI, [*ſee fig. preced.*] divide the ſides *FG*, GH, HI, *IF*, in the middles at the points A, D, B, C, and draw the lines AB and CD, which may cut each other at the point E. I demonſtrate that the lines EA, ED, EB, and EC are equal, and the angles A, D, B, C, right angles: and that therefore you may deſcribe a circle from the center E, which will paſs through A, D, B, and C, and touch the ſides of the ſquare *F*GHI.

Demon-

Demonstration.

Since the lines AB and GH do conjoyn the lines AG and BH, which are parallel and equal, they alſo will be parallel and equal: therefore the figure AGDE is a parallelogram; and the lines AE and GD, AG and ED are equal: and AG and GD being equal, AE and ED will be equal alſo. Tis after the ſame manner that the lines AE, EC, EB, are prov'd equal. Further, AG and CD being parallel, and the angle G a right angle, the angle D will be ſo likewiſe. Therefore the circle ADBC may be deſcrib'd from the center E, which will paſs through the points A, D, B, C, and touch the ſides of the ſquare.

PROPOSITION IX.

A PROBLEM.

To deſcribe a circle about a ſquare.

TO deſcribe a circle about the ſquare ACB D, [*ſee fig. in Prop. 6.*] draw the diagonals *AB*, and CD, which will cut each other at the point E; the point E will be the center of the circle, which will paſs through the points A,C, B,D. It ought therefore to be demonſtrated, that the lines AE, EB, CE, and ED are equal.

Demon-

Demonstration

The sides AC and CB are equal, and the angle C is a right angle; therefore the angles BAC and ABC are equal, (*by the* 5. 1.] and half right angles, (*by the* 32. 1.)

After the same manner I demonstrate, that the angles ACD, ADC, BDC, and BCD, are half right angles. Therefore the triangle AEC having the angles EAC and ECA half right angles, and consequently equal, will have also (*by the* 6. 1.) its sides AE and EC equal. The same may be prov'd of the lines EC and EB, EB and ED, that they likewise are equal.

The USE.

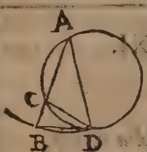
'We show in the 12. book, that Polygons 'inscrib'd in a circle, degenerate into circles: 'and as these Polygons are always in the dupli-'cate proportion of that of their diameters, so 'likewise are circles. In practical *Geometry* 'we have frequent occasion to inscribe a square 'and other Polygons in a circle, or to describe 'them about it, to reduce a circle to a square.

PROP.

PROPOSITION X.

A PROBLEM.

To describe an Isosceles *(or equicrural triangle) having its angles at the base, each of them double to the third angle.*



TO describe an *Isosceles* ABD, having each of its angles ABD and ADB, double the angle A, divide the line AB *(by the* 11.2.) so that the square of AC may be equal to the rectangle under AB and BC; and from the center A at the distance AB describe the circle BD, in which inscribe BD equal to AC; and drawing the line DC describe a circle about the triangle ACD, (*by the* 5.)

Demonstration.

Since the square of AC or BD is equal to the rectangle contain'd under AB and BC, the line BD will touch the circle ACD at the point D, (*by the* 37.3.) therefore the angle BDC will be equal to the angle A, being in the alternate segment CAD, (*by the* 32. 3.) Now the angle BCD, being an external angle in respect of the triangle ACD, is equal to the angles A and

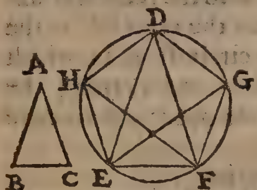
CDA;

CDA; therefore the angle BCD is equal to the angle BDA. Further, the angle ADB is equal to the angle ABD, *(by the 5.1.)* therefore DCB and DBC are equal, and *(by the 6 1.)* the ſides BD and DC will be equal: and ſince BD is equal to AC, the ſides AC and CD will be equal; and ſo likewiſe the angles A and CDA. Therefore the angle ADB is double the angle A.

PROPOSITION XI.

A PROBLEM.

To inſcribe a regular Pentagon in a circle.



TO inſcribe a Regular Pentagon in a circle, deſcribe *(by the* 10.) an *Iſoſceles* ABC, having each of its angles ABC, ACB, at the baſe, double the angle A. Inſcribe in the circle the triangle DEF equiangular to ABC: then divide the angles DEF and DFE into two equal parts, drawing the lines EG and FH. Laſtly, joyn the lines DH, DG, GF, EH, and you will have deſcrib'd a regular Pentagon; that is to ſay, a Pentagon having equal ſides, and equal angles.

f2,4 f9,1

Demon-

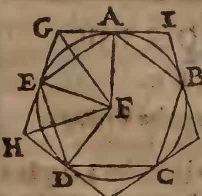
Demonstration.

The angles DEG, GEF, DFH, and HFE, being the halves of the angles DEF and DFE, each of which is double to the angle EDF, are equal to the angle EDF: and consequently the five arches, which are their bases, are equal, (*by the* 26.3.) and the lines DH, HE, EF, FG, and GD, are equal, (*by the* 29.3.) Secondly, the angles DGF, GFE, and so of the rest, having each three of those equal arches for its base, will be also equal, (*by the* 27.3.) Therefore the sides and angles of the Pentagon DHEFG are equal.

PROPOSITION XII.

A PROBLEM.

To describe a Pentagon about a Circle.



INscribe a regular Pentagon ABCDE in the circle, (*by the* 11.) and having drawn tangents through the points A, B, C, D, E, (*by the* 17.3.) you will have describ'd a regular pentagon about the circle. Draw the lines FA, FG, FE, FH, FD.

Demonstration.

The tangents GE and GA are equal, (*by co-*

 roll.

roll. 2 of the 36. 3.) as alſo EH, and HD: the lines FA and FE are alſo equal (*by the definit. of a circle*;) therefore (*by the* 8. 1.) the triangles FGA and FGE are equal in all reſpects; and the angles AFG and EFG are equal, as alſo the angles EFH and DFH. And becauſe (*by the* 27. 3.) the angles EFA and EFD are equal, their halves EFH and EFG will be equal; and (*by the* 26. 1.) the triangles EFH and EFG will be equal in all reſpects; and the ſides EG and EH will be alſo equal. After the ſame manner I can demonſtrate all the ſides to be divided into two equal parts; and conſequently, ſince the lines GE and GA are equal, GH and GI will be alſo equal. Further, the angles G and and H being double the angles FGE, and FHE, are alſo equal, Therefore we have deſcrib'd a regular Pentagon about the circle.

PROPOSITION XIII.

A Problem.

To inſcribe a circle in a regular Pentagon.

TO inſcribe a circle in the regular pentagon ABCDE, divide the angles A and B into two equal parts by the lines AF and BF, which concur at the point

point F. Then drawing the line FG perpendicular to AB, defcribe a circle from the center F at the diftance FG. I fay it will touch all the other fides; that is to fay, having drawn FH perpendicular to BC, FH and FG will be equal.

Demonftration.

Since the equal angles A and B were divided into two equal parts, their halves GAF and GBF will be equal: and fince the angles at G are right angles, the triangles AFG and BFG will be equal in all refpects, (*by the* 26.1.) therefore the lines AG and GB are equal.

Further, I may prove the lines BG and BH, as alfo the lines *FG* and *FH*, to be equal; and the fides AB and BC of a regular pentagon being equal, the lines BH and HC will be equal; and confequently, the angles at the point H being alfo right angles, and equal, the triangles BFH and HFC will be equal in all refpects, and the angles of *F*BH and *F*CH will be equal. And fince the angles B and C are equal, the angle *F*CH will be half the angle C. So paffing from one to the other I will demonftrate, that all the perpendiculars *FG*, and *FH*, and the reft, are equal.

PROPOSITION XIV.

A PROBLEM.

To describe a circle about a regular Pentagon.

TO describe a circle about the regular pentagon ABCDE, divide equally two of its sides AB and BC at G and H, and draw the perpendiculars *GF* and *HF*. The circle drawn from the center *F*, at the distance *FA*, will pass through B, C, D, E.

Demonstration.

Suppose the circle describ'd, it is evident (*by the* 1. 3.) that having divided the line AB in the middle in G, and drawn the perpendicular *GF*, the center of the circle must be in that perpendicular: it is also in *HF*: therefore it is at the point *F*.

The USE.

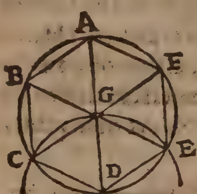
'These Propositions are solely useful for the 'composing the table of Sines, and drawing the 'platforms of Cittadels, for their ordinary fi-'gures are pentagons. Observe also that these 'methods of describing pentagons about a cir-'cle, may be apply'd likewise to other Poly-gons.

'gons. But in my book of Military Archite-'cture, I have shown another way of inscribing 'a regular Pentagon in a circle.

PROPOSITION XV.

A Problem.

To inscribe a regular Hexagon in a Circle.



TO inscribe a regular Hexagon in the circle ABC DEF, draw the diameter AD, and fixing the foot of the compass at the point D describe a circle at the distance of DG: then draw the diameters EGB, and CGF; and the lines AB, *AF*, *FE*, and the rest. *Demonstration.*

Tis evident, that the triangles CDG, and DGE, are equilateral; therefore the angles CGD, DGE, and those oppos'd to them at the top, BGA, and AGF, are each of them the third part of two right angles; that is to say, contain 60 degrees. Now all the angles that can be made about the same point are equal to four right angles, that is to say, 360 degrees. Therefore taking away four times 60, that is 240, from 360, there will remain 120 for BGC and *F*GE; which therefore each contain

60

60 degrees. Therefore all the angles at the center being equal, all the arches and all the ſides will be equal; and every angle as A, B, C, *&c.* will be compounded of two angles of 60 degrees each, that is, 120 degrees, and therefore will be equal.

Coroll. The ſide of a Hexagon is equal to the ſemidiameter.

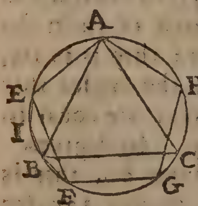
The USE.

'Because the ſide of a Hexagon is the baſe of 'an arch of 60 degrees, and is equal to the ſe-'midiameter, its half will be the ſine of 30; by 'which ſine we begin the table of Sines. *Euclid* 'treats of Hexagons in the laſt book of his *Ele-'ments.*

PROPOSITION XVI.

A PROBLEM.

To inſcribe a regular Pentedecagon in a circle.



INſcribe in the circle an equilateral triangle ABC (*by the* 2.) and a regular pentagon (*by the* 11.) ſo that the angles may meet at the point A. The lines *BF*, BI, and IE, will be the ſides of a Pentedecagon: and if you inſcribe in the other arches

arches lines equal to *BF* and BI, you will compleat the Polygon.

Demonstration.

Since the line AB is the side of an equilateral triangle, the arch AEB will be the third part of the whole circle, that is, five fifteenths, But the arch AE being the fifth part, will contain three fifteenths; therefore the arch EB contains two; and if you divide it in the middle at the point I, each part will be a fifteenth.

The USE.

'This Proposition serves only to open the 'way to other Polygons. We have in the *Compass of Proportion* some most easie methods of 'inscribing all ordinary Polygons, but they 'are grounded on this here. For it would be 'impossible to mark polygons upon that instrument, if their sides were not first found by 'this, or other like propositions.

THE

THE FIFTH BOOK

OF THE

ELEMENTS

OF

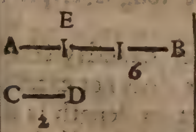
EUCLID.

'THis fifth Book is absolutely necessary to
' demonstrate the propositions of the sixth.
'The doctrine it contains is of universal use;
'and its manner of argumentation by Propor-
'tion the most subtil, solid, and brief. Inso-
'much that all those Treatises, that are ground-
'ed on Proportions, are oblig'd to make use
'thereof, as a kind of *Mathematical Logick.*
'*Geometry*, *Arithmetick*, *Musick*, *Astronomy*,
'*Staticks*, and in a word all parts of *Mathema-*
'*ticks*, are constrain'd to borrow some of their
'demonstrations from the Propositions of this
'Book. The greatest part of Measurings in

pra-

‘ *practical Geometry* is done by Proportion. All ‘ the rules of *Arithmetick* are demonstrable by ‘ the Theorems that occur here; so that there ‘ is no necessity of having recourse to the se‐ ‘ venth, eight, or ninth books for that purpose. ‘ The *Musick* of the Antients is scarce any ‘ thing else but the doctrine of Proportions ap‐ ‘ ply'd to Sounds. The same may be said of ‘ *Staticks*, which considers the proportion of ‘ Weights. In fine tis most certain, that if the ‘ knowledge of Proportions, which this book ‘ affords, was taken away from the *Mathema‐* ‘ *ticks*, what remain'd would be very inconsider‐ ‘ able.

DEFINITIONS.



A Small quantity com‐ par'd with a greater, is call'd a Part. “ As for ‘ example, if the line CD ‘ of two foot long be com‐ ‘ par'd with AB of 6, it will be call'd its *part.* ‘ Which name also it obtains, though indeed it ‘ be not contain'd in AB, provided AE, a line ‘ equal to CD, be therein found.

‘ The whole is answerable to the part, and ‘ therefore will be the greater quantity com‐

'par'd with the less; whether it do really con-
'tain the less, or no.

'A Part taken in general is ordinarily divided 'into (that which is call'd) an *Aliquot* part, and 'an *Aliquant* part.

1. An *Aliquot* part (which alone *Euclid* defines in this book) is a magnitude of a magnitude, a less of a greater, when it exactly measures the greater. "That is to say, 'Tis a lesser 'quantity compar'd with a greater, which pre-'cisely measures the greater. As a line two foot 'long taken three times, is equal to a line of six 'feet in length.

2. A *Multiple* is a magnitude of a magnitude, a greater of a less, when it is exactly mea-sur'd by the less. "That is to say, A Multi-'tiple is a greater quantity compar'd with a 'less, which it contains exactly so many times. 'For example, a line six foot long is triple a 'line two foot long.

An *Aliquant* part, is a lesser quantity compar'd with a greater, which it does not exactly measure. As a line of four feet in length is an aliquant part of a line ten foot long. "In a 'word, An Aliquot part so many times repeat-'ed will equal the whole: but an Aliquant part, 'though it contains such a quantity of the 'whole, yet repeated as you please will never 'exactly equal, but either come short of, or ex-'ceed, the whole.

Equi-

12,	4 ;	6,	2.
A,	B ;	C,	D

Equimultiples are magnitudes which equally contain their aliquot parts. 'That is 'to say, so many times. As 'for example, if A contains B 'as many times as C contains D, A and C will 'be equimultiples of B and D.

3. * *Proportion* is a respect of one magnitude to another of the same kind.

* *Græc.* λόγος. *Gall.* Raison.

4. *Quantities* are said to have a certain proportion to each other, when being multiply'd they can exceed each other. " For which rea'son they ought to be of the same kind. For 'indeed a line has no proportion to a superfi'cies; because a line taken *Mathematically* is 'consider'd without any breadth at all; and 'therefore multiply'd as much as you please 'will never give any breadth, which yet a su'perficies contains.

'For as much as Proportion is a respect or 'relation founded upon quantity, it ought to 'have two terms. That which some Philoso'phers would call the *Fundamentum*, or Foun'dation, *Mathematicians* name the *Antecedent*, 'and the *Term* is call'd by them the *Consequent*. 'As if we were to compare the quantity A with 'the quantity B, that respect or proportion 'would have for the Antecedent the quantity 'A, and for the consequent the quantity B.

'On the contrary, if B be compar'd with A, 'that proportion of B to A would have B for 'the antecedent, and A for the consequent.

'Proportion, or the respect of one quantity 'to another, is divided into Rational Proportion, and Irrational.

Rational Proportion is the respect of one 'quantity to another, which is commensurable 'to it, that is, when both the quantities have the 'same common measure, by which both may be 'exactly measur'd. As the proportion of a line 'four foot long to another that is six, is rational, 'because a line two foot long may exactly measure both. And when this happens, these quantities have the same proportion as one number 'to another. For example, since the line two 'foot long, which is their common measure, is 'found twice in the four-foot line, and thrice 'in that which is six foot long; the first has the 'same proportion to the second, as 2 to 3.

'Irrational Proportion is betwixt two quantities of the same kind, which are incommensurable, *i. e.* have no common measure. As the 'proportion of the side of a square to its diagonal. For there is no measure so small, as will 'precisely measure both.

'Four quantities will be proportionals, when 'the proportion of the first to the second, is the 'same with, or like to, that of the third to the 'fourth; so that, to speak properly, Proportionality

'onality is a similitude of proportions. But it is 'no easie matter to understand in what consists 'this similitude of proportions; that is to say, 'how two respects or relations may be alike. 'For *Euclid* has not given us a just definition 'thereof, or such an one as might explain the 'nature of the thing, but contented himself to 'set down some marks or signs, by which it 'may be known, whether or no quantities had 'the same proportion. And tis the obscurity of 'this definition, which has render'd the whole 'book so difficult to be understood; which de-'fect therefore I shall endeavour to supply.

5. *Euclid* makes four magnitudes to have the 'same proportion, when taking the Equimulti-'ples of the first and the third, and likewise the 'Equimultiples of the second and the fourth, 'according to any multiplication whatsoever: 'if the multiple of the first exceed that of the 'second, the multiple of the third will also ex-'ceed that of the fourth; and if it be equal to, 'or less than the second, the third will be equal 'to or less than the fourth. In such a case the 'first has the same proportion to the second, as 'the third to the fourth.

As

A, B :: C, D	
2, 4 :: 3, 6.	
E, F :: G, H	
10, 8 :: 15, 12	
K, L :: M, N	
8, 8 :: 12, 12	
O, P :: Q, R	
6, 16 :: 9, 24	

'As for example, if four 'magnitudes were propos'd, 'A,B,C,D; having taken 'the Equimultiples of A 'and C, as their quintuples 'E and G; and F and H the 'doubles of B and D; In 'like manner taking K and 'M the quadruples of A and 'C, and L and N the doubles of B and D; A-'gain taking O and Q the triples of A and C, 'and P and R the quadruples of B and D; Be-'cause E being greater than F, G is greater 'than H; and K being equal to L, M is equal 'to N; and lastly, O being less than P, Q is 'less than R: therefore A will have the same 'proportion to B, as C to D. But methinks *Eu-'clid* ought to have demonstrated this proposi-'tion, it being too perplex'd and obscure to pass 'for a principle.

'To explain aright what Proportionality is, 'or how four magnitudes may be in the same 'proportion; though it may be sufficient to say 'in general, that the first ought to be a like 'part, or a like whole in respect of the second; 'as the third is, compar'd with the fourth: yet 'because this definition agrees not to the pro-'portion of Equality, I shall give a more gene-'ral one; and to make it the more intelligible, 'explain first what is a similar or like Aliquot 'part.

Similar

'Similar Aliquot parts, then are such as are 'contain'd in their wholes as many times one 'as the other: as 3 in respect of 9, and 2 in re-'spect of 6, are similar aliquot parts; because they 'are each contain'd three times in their respe-'ctive wholes.

'The first quantity will have the same propor-'tion to the second, as the third to the fourth; 'if the first contains so many times such aliquot 'parts of the second, as the third contains the 'like aliquot parts of the fourth. As A, B, C, D, 'if A contains the hundredth, thou-'sandth, or hundred-thousandth 'part of B, as oft as C contains the hundredth, 'thousandth, or hundred-thousandth part of D; '(and the like may be said of all other aliquot 'parts imaginable;) there will be the same pro-'portion of A to B, as of C to D.

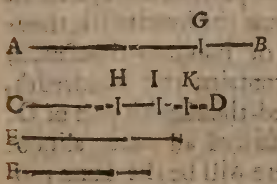
To render this Definition still more clear, I 'will prove first that, if A has the same propor-'tion to B as C to D, A will contain the ali-'quot parts of B, as oft as C does the like of D; 'and secondly, if A contain the aliquot parts of 'B as oft as C does the like of D, then there 'will be the same proportion of A to B, as of 'C to D.

'The first point seems sufficiently evident 'from the very notion of the terms: for if A 'contains the tenth part of B once more than an 'hundred times, and C contains the tenth part

of

'of D an hundred times only ; the quantity A 'will be a greater Whole compar'd to B, than 'C compar'd to D ; therefore it cannot be com-'par'd after the same manner, the respect or re-'lation being not the same.

'The second point seems more difficult, *viz.* if a quantity, suppose AB, contain the aliquot 'parts of another, CD, as oft as a third, E, con-'tains the like of a fourth, F : there will be 'the same proportion of AB to CD, as of E to



'F. But if it be other-'wise, let us suppose AB to have a greater proportion to CD, 'than E has to F; that 'is to say, that AB is 'too great to have the 'same proportion to CD, that E has to F. Therefore a quantity less 'than AB, as AG, will have the same propor-'tion to CD, as E to F. Divide therefore CD in 'the middle in H, and HD in the middle in I, 'and ID in the middle in K; continuing the 'like division till you arrive at an aliquot part 'of CD less than GB, which I will suppose to be KD.

Demonstration.

'Since there is the same proportion of AG 'to CD, as of E to F ; AG will contain KD, 'an aliquot part of CD, as oft as E contains the like

'like aliquot part of F. Now AB will contain 'KD once more than E contains the like aliquot 'part of F; which is contrary to the suppoſi-'tion.

6. The firſt Quantity is ſaid to have a greater proportion to the ſecond, than the third to the fourth, when the firſt contains a certain aliquot part of the ſecond oftner than the third contains the like aliquot part of the fourth: As '101 has a greater proportion to 10 than 200 'to 20, becauſe 101 contains the tenth part of '10, that is, 1, once above a hundred times; and '200 contains the tenth part of 20, *i.e.* 2, a 'hundred times only.

7. Magnitudes or Quantities having the ſame proportion, are called Proportionals.

8. * Proportionality, or Analogy, is a ſimilitude of Proportions or Reſpects.

* Ἀναλογία. *Eucl.*

9. In each proportionality are required at the leaſt three terms. "For that there may be 'a ſimilitude of proportions, there muſt be two 'of them: and every proportion having two 'terms, an Antecedent and a Conſequent, there 'ſeems to be a neceſſity of four terms; as when 'we ſay, that A has the ſame proportion to B 'as C to D; but becauſe the conſequent of the 'firſt proportion may be the antecedent of the 'ſecond, three terms may ſuffice; as when A is 'ſaid to have the ſame proportion to B as B to C.

E e 10. Mag-

10. Magnitudes are said to be continually proportionals, when the intermediate terms are taken twice, *i. e.* both as antecedents and consequents. "As if there be the same proportion of A to B, as of B to C, and of C to D.

11. In that case A will have the duplicate proportion to C, and the triplicate to D, of what it has to B.

'But here it is to be observ'd, that there is a 'great deal of difference between double proportion, and duplicate. We say that the proportion of four to two is double, because four 'is the double of two; the number two giving 'the name to the proportion, or rather to the 'antecedent thereof. Accordingly double, triple, quadruple, quintuple, *&c.* are denominations drawn from the numbers two, three, four, 'five, *&c.* compar'd with unity; which I instance in, because we more easily conceive the 'proportion, the less are its terms. But, as I 'said, these denominations do rather affect the 'antecedents, than the proportions themselves; 'for we call that double or triple proportion, 'whose antecedent is double or triple its consequent. But by duplicate proportion we understand such an one, as is compounded of two 'similar proportions. As, if there be the same 'proportion of two to four, as of four to eight: 'the proportion of two to eight being compounded of the proportion of two to four, and

that

'that of four to eight, which are ſimilar and 'equal, will be the duplicate of each of them. 'So three to twenty ſeven is the duplicate pro-'portion of that of three to nine. The propor-'tion of two to four is call'd the ſub-double, be-'cauſe two is the half of four; but that of two 'to eight is the duplicate of the ſub-double; 'which is as much as to ſay, that two is the half 'of half of eight, as three is the third part of 'the third part of twenty ſeven; where you 'may obſerve, that the Denominators $\frac{1}{2}$ and $\frac{1}{3}$ are taken twice.

'In like manner the proportion of eight to 'two is a duplicate proportion of that of eight 'to four, becauſe eight is the double of four, 'but it is the double of the double of two. If 'there be four terms in continual proportion, 'the proportion of the firſt to the laſt is a tripli-'cate of that of the firſt to the ſecond; as in 'theſe four numbers, Two, Four, Eight, Six-'teen; the proportion of two to ſixteen is a tri-'plicate of that of two to four, becauſe two is 'the half of the half of the half of ſixteen. So a'-'ſo the proportion of ſixteen to two is a tripli-'cate of that of ſixteen to eight, becauſe ſixteen 'is the double of the double of the double of 'two.

12. Antecedents to antecedents, and conſe-'quents to conſequents, are call'd Homologous 'magnitudes. As if there be the ſame proportion

'of A to B, as of C to D; A and C, B and D, 'are homologous.

' The following Definitions explain the divers 'manners of arguing from proportionals: for the 'demonſtration of which this book was princi- 'pally compos'd.

13. Alternate proportion is when we compare the antecedent of the one with the antecedent of the other, and the conſequent of the one with the conſequent of the other. " As for 'example, if becauſe there is the ſame propor- 'tion of A to B, as of C to D, I infer, that there 'is the ſame proportion of A to C, as of B to D. 'This manner of argumentation holds only when 'all the four terms are of the ſame *ſpecies* or 'kind; *i.e.* either all lines, or all ſuperficies's, 'or all ſolids. Tis demonſtrated *Prop.*16.

14.*Inverted Proportion is the comparing of 'the conſequents with the antecedents.

*Ἀνάπαλιν *Eucl.* Converſe. *Gall.*

'As, if becauſe there is the ſame proportion ' of A to B, as of C to D, I conclude that there 'is the ſame proportion of B to A, as of D to 'C. *Corol. of Propoſ.* 16.

15. Compoſition of Proportion is the comparing of the antecedent and the conſequent taken together, with the conſequent alone. 'As if, becauſe there is the ſame proportion of 'A to B, as of C to D, I conclude that there is 'the ſame proportion of A and B to B, as of C 'and D to D. *Prop.* 18.

16. Di-

16, Division of Proportion is the comparing of the Excess of the antecedent above the consequent to the same consequent. "As, if there 'be the same proportion of AB to B, as of CD 'to D; from thence I infer, that there is the 'same proportion of A to B, as of C to D. *Proposition* 17.

17. Conversion of Proportion is the comparing of the antecedent with the Difference of the Terms: "As, if there be the same pro-'portion of AB to B, as of CD to D. I thence 'conclude, that there is the same proportion of 'AB to A, as of CD to C. *Coroll. of Prop.* 18.

18. Proportion of Equality is the comparing of the extreme magnitudes, and omiting those in the middle. "As if, there be-'ing the same proportion of A to 'to B, as of E to F; and of B to 'C, as of F to G; and of C to D, 'as of G to H; I infer, that there is the same 'proportion of A to D, as of E to H.

A.B.C.D.
E.F.G.H.

19. Proportionality of Equality orderly plac'd, is that in which the terms are compar'd together in the same order. "As in the fore-'going example. *Prop.* 22.

20. Proportionality of Equality disorderly plac'd, is that in which the terms are compar'd in a different order. "As if, there being the 'same proportion of A to B, as of G to H; and 'of B to C, as of F to G; and of C to D, as of

E

'E to F; I conclude that there is the ſame pro-
'portion of A to D, as of E to H. *Prop.* 23.

'See in ſhort all the different manners of ar-
'gumentation by Proportion.

'As A to B, ſo C to D; therefore
'By Alternate proportion: as A to C, ſo B
'to D:
'Inverted: As B to A, ſo D to C:
'Compoſition. As AB to B, ſo CD to D.
'Diviſion. If as AB to B, ſo CD to D: then
As A to B, ſo C to D.
'Converſion. As AB to A, ſo CD to C.
Orderly Equality. If as A to B, ſo C to D; and
as B to E, ſo D to F: then
as A to E, ſo C to F.
Diſorderly Equality. If as A to B, ſo D to F, and
as B to E, ſo C to D: then
as A to E, ſo C to F.

'*Euclid's* fifth book contains but 25 propoſi-
'tions, to which nine more have ſince been add-
'ed, and are commonly receiv'd. And the firſt
'ſix in *Euclid*, ſerving only for the proof of
'thoſe that follow by the method of *Equimul-*
'*tiples*, ſince I intend not to make uſe of that me-
'thod, I ſhall wholly omit; begining with the Se-
'venth, without changing either the order or
'number of the Propoſitions.

Demands

Demands, or Suppositions.

Three quantities A, B, C, being propos'd, it is requir'd to be granted, that there is a fourth possible, to which the quantity C has the same proportion, as A to B.

PROPOSITION VII.

A THEOREM.

Equal quantities have the same proportion to another quantity, and another quantity has the same proportion to quantities that are equal.

A, 8.
C, 4.
B, 8.

If the quantities A and B be equal, they will have the same proportion to the third C.

Demonstration.

If one of the two, suppose A, had a greater proportion to C than B: A would contain any aliquot part of C, oftner than B could contain the same; and consequently A would be greater than B, which is contrary to what was suppos'd.

Again, I say, if the quantities A and B be equal, the quantity C will have the same proportion to A as to B.

If

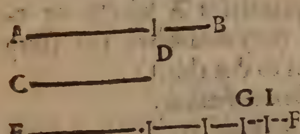
Demonſtration.

If the quantity C had a greater proportion to A, than to B, it would contain a certain aliquot part of A, oftner than the like part of B; which part therefore of A muſt be leſs than the like aliquot part of B, and conſequently the quantity A would be leſs than B, which is contrary to the ſuppoſition.

PROPOSITION VIII.

A Theorem.

The greater of two quantities has a greater proportion to the ſame, than the leſs; and the ſame quantity has a leſſer proportion to the greater, than to the leſs.



Suppoſe the quantities AB and C be compar'd with the ſame EF, and that AB exceeds C. I ſay, that AB will have a greater proportion to EF, than C will have to the ſame. Cut AD equal to C, and divide EF in the middle, and again one half in the middle, and ſo on, till you come to an aliquot part of EF leſs than DB, as GF.

Demon-

Demonstration.

AD and C being equal, AD has the same proportion to EF, as C to EF, (*by the* 7.) and therefore AD will contain GF an aliquot part of EF, as oft as C will contain the same, (*by defin.* 5.) But AB contains the same aliquot part once more than AD, DB being greater than GF; therefore (*by defin.* 6.) the proportion of AB to EF is greater than that of C to the same EF.

Secondly, I say, that EF has a less proportion to AB, than to C. Take a certain aliquot part, as the fourth, of C, as oft as you can in EF, which suppose to be five times; either there will remain something of the quantity EF, or nothing; if nothing remain, it is evident, that five times the fourth part of AB making a greater line than so many times the fourth part of C, the fourth part of AB could not be five times contain'd in EF. But if the fourth part of C taken five times reach no farther than G, the fourth part of AB taken so many times will proceed either as far as F, or to I something short of F. If it reach as far as F, EF will have the same proportion to AB as EG to C. But (*by the preceding part*) EF has a greater proportion to C, than EG to the same C; therefore EF has greater proportion to C, than to AB. But if the fourth part of AB reach no farther than I, EI will have the same proportion to AB as EG to

C. But EI has a greater proportion to C, than EG to C; therefore EF, greater than EI, has a greater proportion to C, than the same EF to AB.

PROPOSITION IX.

A THEOREM.

Quantities that have the ſame proportion to another quantity, or to whom another quantity has the ſame proportion, are equal,

A, B, C,
12.12.6

If the quantities A and B have the ſame proportion to a third quantity C, I ſay, A and B are equal.

Demonſtration,

If one of the two, *v. g.* A were greater than B, it would have a greater proportion to the quantity C, (*by the* 8.) which is contrary to the ſuppoſition.

Secondly, if the quantity C has the ſame proportion to the quantities A and B; I ſay A and B are equal. For if A were greater than B, C would have a greater proportion to the quantity B, than to A, (*by the* 8.) which is alſo contrary to the ſuppoſition.

PROP.

PROPOSITION X.

A Theorem

The quantity that has the greater proportion to another, is the greater quantity; and that the lesser, to which that other quantity has the greater proportion.

A.B.C

If the quantity A has a greater proportion to the quantity C, than B to the same C, I say, A is greater than B. For if A and B were equal, they would have both the same proportion to C; and if A were less than 7 B, B would have a greater proportion to C, than 8 A to the same C; both which are contrary to the supposition.

Secondly, if C has a greater proportion to B than to A, I say that A will be greater than B. For if A and B were equal, C would have the same proportion to both, (*by the* 7.) And if A were less than B, C would have a less proportion to B than to A, (*by the* 8.) both which are contrary to what was suppos'd.

PROPOSITION XI.

A THEOREM.

Proportions that are equal to another, are also equal amongst themselves.

A,	B;	C,	D;	E	F.
4,	2;	8,	4;	6,	3.

IF A has the same proportion to B, as C to D; and C the same proportion to D, as E to F; I say, A will have the same proportion to B, as E to F.

Demonstration.

Since A has the same proportion to B, as C to D; A will contain any certain aliquot part of B, as oft as C contains the like aliquot part of D, (*by defin.* 5.) And in like manner, as oft as C contains that aliquot part of D, so oft will E contain the like aliquot part of F. So that as oft as A contains any certain aliquot part of B, so many times also will E contain the like aliquot part of F. Therefore A has the same proportion to B, as E to F.

PROP.

PROPOSITION XII.

A THEOREM.

If many quantities be proportional, one Antecedent will have the same proportion to his consequent, as all the Antecedents taken together to all the Consequents.

A,	B,
3,	12,
C,	D,
2,	8,

IF A has the fame proportion to B, as C to D; I fay that A and C taken together will have the fame proportion to B and D, as A to B.

Demonftration.

Since A has the fame proportion to B, as C to D; the quantity A will contain any certain aliquot part of B, as oft as C contains the like aliquot part of D, (*by defin.* 5.) fuppofe the fourth part. Now the fourth part of B and the fourth part of D, make the fourth part of BD; and accordingly AC will contain the fourth part of BD, as oft as A contains the fourth part of B; and the like may be faid of any other aliquot parts. Therefore A has the fame proportion to B, as AC to BD.

PROP.

PROPOSITION XIII.

A Theorem.

If of two equal proportions one is greater than a third, the other will be ſo likewiſe.

A, B : C, D : E, F.

IF A has the ſame proportion to B, as C to D; but a greater proportion to B, than E to F: I ſay, that C alſo will have a greater proportion to D, than E to F.

Demonſtration.

Since A has a greater proportion to B than E to F, A will contain a certain aliquot part of B, oftner than E contains the like aliquot part of F, (*by defin. 6.*) but C contains a like aliquot part of D, as oft as A contains that of B; becauſe A has the ſame proportion to B, as C to D: and therefore C contains a certain aliquot part of D, oftner than E contains the like aliquot part of F; and conſequently, C has a greater proportion to D, than E to F, (*by def. 6.*)

PROP.

PROPOSITION XIV.

A THEOREM.

If the firſt quantity has the ſame proportion to the ſecond, as the third to the fourth; according as the firſt is greater, or equal, or leſs, than the third, the ſecond will be greater, or equal, or leſs, than the fourth.

A, B; C, D.

IF A has the ſame proportion to B, as C to D; I ſay firſt, if A be greater than C, B will be alſo greater than D.

Demonſtration.

Since A is greater than C, A will have a greater proportion to B, than C to the ſame B, (*by the* 8.). But there is the ſame proportion of A to B, as of C to D: therefore C has a greater proportion to D, than C to B, and conſequently (*by the* 10.) B is greater than D.

Secondly, if A be equal to C, B will be alſo equal to D.

Demonſtration.

Since A and C are equal, there will be the ſame proportion of A to B, as of C to the ſame B, (*by the* 7.) But as A to B, ſo C to D; therefore C has the ſame proportion to B, as the ſame C to D, and conſequently B and D are equal, (*by the* 9.)

Third-

Thirdly, if A be leſs than C, B will alſo be leſs than D. *Demonſtration.*

Since A is leſs than C, A will have a leſs proportion to B, than C to the ſame B, *(by the* 8.) But as A is to B, ſo C to D: therefore C will have a leſs proportion to D, than the ſame C to B, and conſequently B will be leſs than D, (*by the* 10.)

PROPOSITION XV.

A THEOREM.

Equimultiples, and ſimilar aliquot parts, are in the ſame proportion.

A,B;C,D,
2,3;6,9.
E,2;H,3,
F,2;I,3,
G,2,K,3

IF the quantities C and D be the Equimultiples of A and B, their aliquot parts, A will have the ſame proportion to B, as C to D. Divide the quantity C into parts equal to A, *v. g.* E, F, G, and the quantity D into parts equal to B. Becauſe C and D are the equimultiples of A and B, there will be as many parts of one, as of the other. Let the parts of D therefore be H,I,K.

Demonſtration.

E has the ſame proportion to H, and F to I, and G to K, as A to B, becauſe they are all equal.

qual. Therefore (*by the* 12.) as A to B, ſo E, F, G, to H, I, K, *i. e.* ſo C to D.

Coroll. The ſame numbers of the aliquot parts of two quantities are in the ſame proportion that the quantities are. For ſince E has the ſame proportion to H, as C to D; and F to I, as C to D; E and F will have the ſame proportion to H and I, as C to D.

PROPOSITION XVI.

A Theorem.

Alternate Proportion.

If four magnitudes of the ſame kind be proportional, they will be alſo alternatively ſo.

A,	B;	C,	D,
12,	8;	9,	6,

IF A has the ſame proportion to B as C to D, and all the four quantities are of the ſame kind, that is, either all Lines, or all Superficies's, or all Solids; A will have the ſame proportion to C, as B to D. For if not, ſuppoſe A to have a greater proportion to C, than B to D.

Demonſtration.

Since tis ſuppos'd, that A has a greater proportion to C than B to D, the quantity A will contain a certain aliquot part of C, *v. g.* a third

part, oftner than B contains a third part of D. Let A therefore contain a third part of C four times, but B the third part of D only three times; having then divided A into four parts, each will contain one third part of C; but B being divided into four parts, they will not contain each of them a third part of D; therefore three fourths of A will contain three thirds of C, that is, the whole quantity of C; but three fourths of B will not contain three thirds, or the whole quantity of D. But on the contrary, since there is the same proportion of A to B, as of C to D, there will be also the same proportion of three fourths of A to three fourths of B, as of C to D, (*by the coroll. of the* 15.) and (*by the* 14.) if three fourths of A be equal to C, three fourths of B will be equal to D; therefore A cannot have a greater proportion to C, than B to D.

A LEMMA.

If the first have the same proportion to the second, as the third to the fourth; any aliquot part of the first will have the same proportion to the second, as the like aliquot part of the third to the fourth.

16,	3;	32,	6,
A,	B;	C,	D,
E,		F.	
4,		8.	

If A has the same proportion to B, as C to D; and E be an aliquot part of A, and F a like aliquot part of C:

C: I say, that E vill have the same proportion to B, as F to D.

Demonstration.

If E had a greater proportion to B than F to D, E would contain a certain aliquot part of B, oftner than F contains the like aliquot part of D; and consequently, E taken twice, thrice, or four times, would contain an aliquot part of B oftner than F taken twice, thrice, or four times contain'd the like aliquot part of D. But E taken four times is equal to A; and likewise F taken four times is equal to C; therefore A would contain an aliquot part of B oftner than C contain'd the like aliquot part of D, and by consequence A would have a greater proportion to B than C to D; which is contrary to the supposition.

A COROLLARY,

" which *Euclid* places after his *4th* Proposition.

Inverted Proportion.

If the first has the same proportion to the second, as the third to the fourth; the second will have the same proportion to the first, as the fourth to the third.

A,	B;	C,	D,
4,	8;	12,	24,
E,		F,	
1,		3.	

If A has the same proportion to B as C to D, B will have the same proportion to A as D to C.

Demonstration.

If B had a greater proportion to A than D to C, B would contain an aliquot part of A, suppose a fourth E, oftner than D contain'd F the fourth part of C. Let us suppose then that B contains eight times the quantity E; D must contain but seven times the quantity F. Now since A has the same proportion to B as C to D, E will have the same proportion to B as F to D, (*by the preceding Lemma,*) and (*by the* 15.) E taken eight times will have the same proportion to B, as F taken eight times to D; but E taken eight times is contain'd in B, therefore F taken eight times must be contain'd in D, notwithstanding what was shown to the contrary; therefore B cannot have a greater proportion to A, than D to C.

The USE.

'The Followers of *Averroes* seem to have 'made use of a manner of argumentation not 'much unlike this, to prove, that *the world had 'existed from eternity*; urging, that there is the 'same proportion between an eternal act of the 'will of God, and the eternal production of the 'world, as between a temporal act, and a tem-'poral effect; therefore by a kind of Alterna-'tion, there is the same proportion of a tempo-'ral act of the will, *i. e.* an act begining in time, 'to an eternal effect; as of an eternal will to a 'temporal effect. Now tis evident, that the

will

‘will, or an act of the will that begins in time, cannot produce an eternal effect; therefore the eternal act of God could not produce an Effect in time. But this argument is faulty in two respects; first, in that it supposes it possible for an act of the Divine will to begin in time; and secondly, in that it is drawn from Alternate proportion, though the terms be of a different kind or *species*.

PROPOSITION XVII.

A Theorem.

The Division of Proportion.

If compounded quantities be proportional, they will be so likewise being divided.

AB,	B;	CD,	D,
8,	3;	16,	6,
A,	B;	C,	D,
5,	3:	10,	6.

IF AB has the same proportion to B as CD to D, A will have the same proportion to B, as C to D.

Demonstration.

Since AB is suppos'd to have the same proportion to B as CD to D, AB will contain a certain aliquot part of B, as oft as CD contains the like aliquot part of D. Now that aliquot part must be found as oft in B, as the like aliquot part is found

found in D. Therefore taking away B from AB, and D from CD, A will contain as many aliquot parts of B, as C contains the like of D, and consequently A will have the same proportion to B, as C to D.

PROPOSITION XVIII.

A Theorem.

The Composition of Proportion.

If quantities, being divided, be proportional, they will be so likewise when compounded.

A,	B;	C,	D,
5,	3;	10,	6
AB,	B;	CD,	D,
8,	3;	16,	6,

If A has the same proportion to B as C to D, AB also will have the same proportion to B as CD to D.

Demonstration.

Since A is suppos'd to have the same proportion to B as C to D, A will contain any aliquot part of B, as oft as C contains the like aliquot part of D. Now the quantity B contains any of its own aliquot parts, as oft as D contains the like of his; therefore adding B to A, and D to C, AB will contain any aliquot part of B as oft as D contains the like aliquot part of D, and consequently (*by defin.* 5.) AB will have the same proportion to B as CD to D.

A

A COROLLARY.

The Conversion of Proportion.

If AB has the ſame proportion to B as CD to D, then AB will have the ſame proportion to A, as CD to C. For (*by the preceding*) A has the ſame proportion to B, as C to D: and (*by the Coroll. of the* 16.) B will have the ſame proportion to A, as D to C; and therefore compounding them, AB will have the ſame proportion to A, as CD to C.

The USE.

“ We have frequent uſe of this manner of
“ argumentation in almoſt all parts of the *Ma-*
“ *thematicks*.

PROPOSITION XIX.

A THEOREM.

If the Wholes be in the ſame proportion, as the parts that are taken away from them, the Remainders will be alſo in the ſame proportion.

AB,	CD ;	B ,	D ,		
16 ,	8 ;	4 ,	2 ,		
A ,	C ;	AB,	CD,		
12 ,	6 ;	16 ,	8 ,		

IF the quantity AB has the ſame proportion to CD, as the part B to the part D: I ſay, A will have

have the ſame proportion to C, as AB to CD,

Demonſtration.

AB is ſuppos'd to have the ſame proportion to CD, as B to D: therefore alternatively (*according to the* 16.) AB has the ſame proportion to B, as CD to D; and by converſion of proportion, AB will have the ſame proportion to A, as CD to C; and again alternatively, there will be the ſame proportion of AB to CD, as of A to C.

The USE.

'This Propoſition is commonly made uſe of 'in the rule of Fellowſhip. For inſtead of work'ing by the rule of Three for every particular 'Aſſociate or Partner, having done it for the 'reſt, to the laſt they aſſign the Remainder of 'the Gain; ſuppoſing that if there be the ſame 'proportion of the whole ſumme of all the Prin'cipals to the whole Gain, as of the Principal 'of one Aſſociate to his part of the Gain: there 'will be alſo the ſame proportion of the Princi'pal that remains to the Remainder of the 'Gain.

'The 20 and 21 Propoſitions are not neceſ'ſary.

PROP.

PROPOSITION XXII.

A Theorem.

The proportion of Equality orderly plac'd.

If divers terms be propos'd, and an equal number of others compar'd with them, so that those which answer to each other in the same order be proportional; the firsts and the lasts will be also proportional.

12,	6,	2 ;	6,	3,	1,
A,	B,	C;	D,	E,	F.

IF the quantities A,B,C, and the quantities D,E,F be proportional; that is, if there be the same proportion of A to B as of D to E, and of B to C as of E to F; A will also have the same proportion to C, as D to F.

Demonstration.

If A has a greater proportion to C than D to F, A will contain an aliquot part, *v.g.* the half of C, oftner than D can contain the half of F. Let us suppose then the half of C to be contain'd twelve times in A, and the half of F only eleven times in D. Now because B has the same proportion to C as E to F, the quantity B will contain the half of C, as oft as E contains the half of F: Suppose then those halves to be con-

tain'd ſix times in each, B and E. A, which contains the half of C twelve times, will have a greater proportion to B, which contains the ſame half of C ſix times; than D, which contains the half of F eleven times only, to E, which contains the ſame half of F ſix times; and conſequently A will have a greater proportion to B, than D to E, which is contrary to what was ſuppos'd.

PROPOSITION XXIII.

A THEOREM

The proportion of Equality diſorderly plac'd.

If two Orders of terms, be in the ſame proportion, diſorderly plac'd: the firſt and laſt of both Orders will be proportional.

A,	B,	C.	D,	E,	F,	G,
12,	6,	3.	8,	4,	2.	1,

IF the quantities A, B, C, and the others D, E, F, equal in number, be in the ſame proportion, diſorderly plac'd, that is, if A has the ſame proportion to B as E to F, and B to C as D to E: A will have the ſame proportion to C, as D to F. Suppoſe B to have the ſame proportion to G, as F to G.

Demon-

Demonſtration,

Since there is the ſame proportion of A to B as of E to F, and of B to C as of F to G; A has the ſame proportion to C, as E to G, *(by the 22.)* Further, ſince B has the ſame proportion to C as D to E, and alſo as F to G; D will have the ſame proportion to E, as F to G, *(by the 11.)* and alternatively,*(according to the 16.)* D will have the ſame proportion to F, as E to G. Now we have before prov'd, that as E to G, ſo A to C; therefore, as A to C, ſo D to F.

PROPOSITION XXIV.

A Theorem.

If the firſt quantity has the ſame proportion to the ſecond, as the third to the fourth; and alſo the fifth to the ſecond, as the ſixth to the fourth: the firſt with the fifth will have the ſame proportion to the ſecond, as the third with the ſixth to the fourth.

E,		F.	
4,		6.	
6,	2 ;	9,	3 ,
A,	B;	C,	D,

IF A has the ſame proportion to B as C to D, and E to B as F to D; AE will have the ſame proportion to B, as CF to D.

Demonſtration.

Since A has the ſame proportion to B as C to

D, A will contain any aliquot part of B, as oft as C contains the like aliquot part of D, (*by defin.* 5.) In like manner, E will contain the ſame aliquot part of B, as oft as F contains the like of D; ſo that A and E will contain any aliquot part of B, as oft as C and F contain the like aliquot part of D: therefore AE will have the ſame proportion to B, as CF to D.

PROPOSITION XXV.

A Theorem.

If four magnitudes be proportional, the greateſt and the leaſt will exceed the other two.

B,	D,			
8,	6,			
4,	3;	4	3,	
A,	C;	E,	F,	

IF the four magnitudes AB, CD, E, F, be proportional, and AB the greateſt and F the leaſt; AB and F will exceed CD and E. Since AB has the ſame proportion to CD as E to F, and AB is ſuppos'd greater than E; CD will be alſo greater than F, (*by the* 14.) Divide therefore AB ſo, that the magnitude A may be equal to E; and CD ſo, that the magnitude C may be equal to F.

Demonſtration.

Since AB has the ſame proportion to CD as A to C, B will alſo have the ſame proportion to D as AB to CD, (*by the* 19.) and AB being ſup-

suppos'd greater than CD, B will be greater than D. Now if to A and E, which are equal, be added F and C, which are also equal, A and F will be equal to C and E, and adding to the two first B, which is the greater, and to the two last D, which is the less, AB and F will be greater than CD and E.

The USE.

By this Proposition is demonstrated a pro-'priety of *Geometrical* proportionality, whereby 'it is distinguish'd from that which is call'd '*Arithmetical.* For in this latter the two mid-'dle terms are equal to the two extreams; but 'in the former, (as has been prov'd,) the great-'est and the least exceed the two others.

'Tho the nine following Propositions are not '*Euclid*'s; yet, because many make use of them, 'and quote them as if they were his, I thought 'I ought not to omit them.

PROP.

PROPOSITION XXVI.

A THEOREM.

If the firſt has a greater proportion to the ſecond than the third to the fourth, the fourth will have a greater proportion to the third than the ſecond to the firſt.

9,	4;	6,	3,
A,	B;	C,	D,
E.			
8.			

IF A has a greater proportion to B than C to D, D will have a greater proportion to C than B to A. Suppoſe E to have the ſame proportion to B as C to D, A will be greater than E, (*by the* 10.)

Demonſtration.

There is the ſame proportion of E to B, as of C to D: therefore (*by the Coroll. of the* 16.) D has the ſame proportion to C, as B to E. But B has a greater proportion to E than to A, (*by the* 8.) therefore D has a greater proportion to C, than B to A.

PROP.

PROPOSITION XXVII.

A THEOREM.

If the first has a greater proportion to the second than the third to the fourth, the first will also have a greater proportion to the third than the second to the fourth.

9,	4;	6,	3,
A,	B;	C,	D,
E.			
8.			

IF A has a greater proportion to B than C to D, I say, that A will have a greater proportion to C than B to D. Let E have the same proportion to B, as C to D; in that case A must be greater than E.

Demonstration.

E has the same proportion to B, as C to D: therefore (*by the* 16.) E has the same proportion to C, as B to D. And because A is greater than E, the proportion of A to C will be greater than that of E to C. Therefore the proportion of A to C, is greater than that of B to D.

PROP.

PROPOSITION XXVIII.

A THEOREM.

If the firſt has a greater proportion to the ſecond than the third to the fourth, the firſt and the ſecond will have a greater proportion to the ſecond, than the third and the fourth to the fourth.

9, 4;	6, 3,
A, B;	C, D,
E,	
8.	

IF the proportion of A to B be greater than that of C to D, the proportion of AB to B will alſo be greater than that of CD to D. Suppoſe E to have the ſame proportion to B, as C to D.

Demonſtration.

E has the ſame proportion to B, as C to D: therefore (*by the* 18.) EB has the ſame proportion to B, as CD to D. And AB being greater than EB, AB will have a greater proportion to B, than EB to the ſame B, and conſequently than CD to D.

x 28 x 1

PROP.

PROPOSITION XXIX.

A THEOREM.

If the firſt with the ſecond has a greater proportion to the ſecond, than the third with the fourth to the fourth; the firſt will have a greater proportion to the ſecond, than the third to the fourth.

9,	4;	6,	3.
A,	B;	C,	D,
E.			
8.			

IF the proportion of AB to B be greater than the proportion of CD to D, the proportion of A to B will be alſo greater than that of C to D. Suppoſe EB to have the ſame proportion to B, as CD to D: EB will be leſs than AB, and E leſs than A.

Demonſtration.

Since EB has the ſame proportion to B, as CD to D; dividing them E will have the ſame proportion, to B as C to D, (*by the* 17.) And A being greater than E, the proportion of A to B will be greater than that of E to the ſame B; and conſequently than that of C to D.

PROPOSITION XXX.

A Theorem.

If the proportion of the first with the second to the second, be greater than that of the third with the fourth to the fourth; the proportion of the first with the second to the first, will be less than that of the third with the fourth to the third.

A,	B;	C,	D,
9,	4;	6,	3.

If the proportion of AB to B be greater than that of CD to D, the proportion of AB to A will be less than that of CD to C.

Demonstration.

The proportion of AB to B is suppos'd to be greater than that of CD to D: therefore (*by the 29.*) the proportion of A to B will be greater than that of C to D; and (*by the 26.*) the proportion of D to C will be greater than that of B to A: therefore being compounded (*by the 28.*) the proportion of CD to C, will be greater than that of AB to A.

PROP.

PROPOSITION XXXI.

A THEOREM.

If many quantities are in a greater proportion among themselves, than an equal number of other quantities, plac'd after the same manner; the first of the first order will be in a greater proportion to the last of that order, than the first of the second order to the last of that.

A, B, C,	D, E, F,
16, 10, 3,	9, 6, 2,

IF A has a greater proportion to B, than D to E; and B a greater proportion to C, than E to F; A will have a greater proportion to C, than D to F.

Demonstration.

Since A has a greater proportion to B than D to E, A will alſo have a greater proportion to D than B to E; and becauſe B has a greater proportion to C than E to F, B will alſo have a greater proportion to E than C to F. Therefore A will have a greater proportion to D than C to F: and Alternatively (*by the* 27.) A will have a greater proportion to C, than D to F.

PROPOSITION XXXII.

A THEOREM.

If many quantities are in greater proportion among themſelves, than an equal number of other quantities, plac'd after a different manner; the firſt of the firſt order will have a greater proportion to the laſt of that order, than the firſt of the ſecond order to the laſt of that.

13,	6,	2,			
A,	C,	E,			
B,		F	H,	I,	K,
12,		3	4,	2,	1,

IF A has a greater proportion to C than I to K, and C a greater proportion to E than H to I; A will have a greater proportion to E, than H to K. Suppoſe B to have the ſame proportion to C as I to K, and C the ſame proportion to F as H to I; then A will be greater than B, and F than E.

Demonſtration.

Since tis ſuppos'd that B has the ſame proportion to C as I to K, and C to F as H to I; B will have the ſame proportion to F, as H to K, (*by the* 23.) But A has a greater proportion to F, than B to the ſame F, (*by the* 8.) and the proportion of A to E is greater than that of A to F, becauſe F is greater than E: therefore the proportion of A to E, is greater than that of H to K.

PROP.

PROPOSITION XXXIII.

A THEOREM.

If the Whole has a greater proportion to the Whole than the part taken away to the part taken away, the Remainder will have a greater proportion to the Remainder than the Whole to the Whole.

13,	4;	6,	2.
A,	B;	C,	D.

IF AB has a greater proportion to CD than B to D, A will have a greater proportion to C than AB to CD.

Demonstration.

We suppose that the proportion of AB to CD is greater than that of B to D; therefore (*by the* 27.) the proportion of AB to B is greater than that of CD to D: and (*by the* 30.) the proportion of AB to A, is less than that of CD to C; therefore alternatively, the proportion of AB to CD is less then that of A to C.

PROP.

PROPOSITION XXXIV.

A THEOREM.

If two orders of magnitudes be propos'd, and the proportion of the first of the first, to the first of the second, be greater than that of the second to the second; and that greater than that of the third to the third, &c. the whole first order will have a greater proportion to the whole second, than the whole first order except its first magnitude, to the whole second order except its first magnitude. But a less proportion than that of the first magnitude of the first order, to the first magnitude of the second; and lastly, a greater proportion, than that of the last magnitude of the first order, to the last of the second.

12, 6, 4,	4, 2, 3,
A, B, C,	E, F, G,

IF the proportion of A to E be greater than that of B to F, and the proportion of B to F greater than that of C to G: I say, that A, B, C, will have a greater proportion to E,F,G, than the proportion of BC to FG.

Demonstration.

Tis suppos'd the proportion of A to E is greater than that of B to F; and therefore alternatively

ternatively, the proportion of A to B is greater than that of E to F; and by compounding them, the proportion of AB to B greater than that of EF to F; and again alternatively, the proportion of AB to EF greater than that of B to F. And becauſe the proportion of the whole AB to EF is greater than that of the part B to the part F, the proportion of the Remainder A to the Remainder E will be greater than that of the whole AB to the whole EF. In like manner, I may prove the proportion of B to F greater than that of BC to FG, and conſequently that of A to E much greater than that of BC to FG. Therefore alternatively, the proportion of A to BC is greater than that of E to FG; and compounding them, the proportion of A,B,C, to BC, greater than that of E,F,G, to FG: therefore alternat: the proportion of A, B, C, to E,F,G, will be greater than that of BC to FG.

Secondly, the proportion of A to E, is greater than that of A,B,C, to E,F,G.

Demonſtration.

I have demonſtrated, that the proportion of the whole A,B,C, to the whole E,F,G, is greater than that of the part BC to the part FG: therefore the proportion of the Remainder A to the Remainder E, will be greater than that of the whole A,B,C, to the whole E, F, G, (*by the* 33.)

Thirdly, the proportion of A, B, C, to E,F,G, is greater than that of C to G, *De-*

Demonstration.

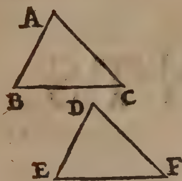
The proportion of A to E is greater than that of B to F; and therefore alternatively, that of A to B is greater than that of E to F; and compounding them, the proportion of AB to B will be greater than that of EF to F; and again alternatively, that of AB to EF greater than that of B to F. But the proportion of B to F is greater than that of C to G, therefore the proportion of AB to E*F* is greater than that of C to G; and that of AB to C greater than that of *EF* to G; and therefore by compounding them, the proportion of A,B,C, to G, will be greater than that of E, *F*, G, to G; and that of A,B,C, to E, *F*,G, greater than that of C to G.

THE

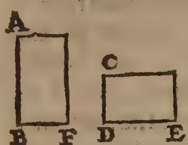
THE SIXTH BOOK
OF THE
ELEMENTS
OF
EUCLID.

THis Book begins to apply the doctrine of Proportions, explain'd in general in the 'preceding, to particular matters; and taking 'its rise from the most simple figures, *i. e.* Tri-'angles, it gives Rules to determine not only 'the proportion of their sides, but also that of 'their capacity, *Area* or superficies. In the 'next place we learn from it, how to find out 'proportional lines, and to augment or diminish 'any figure, according to any proportion assign'd. 'Here also is demonstrated the most useful Rule 'of Three; and the Forty-seventh of the First 'extended to any figure whatsoever. Lastly, it 'lays down the most facile and most certain 'principles to conduct us in taking any manner of Dimensions.

DEFINITIONS.



1. RECtilineal figures are similar, when their angles are equal, and the sides, that form those angles, proportional. " As the triangles 'ABC, DEF, will be similar, 'if the angles A and D, B and 'E, C and F, be equal; and AB has the same 'proportion to AC as DE to DF, and AB to 'BC as DE to EF.

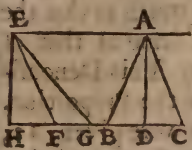


2. Figures are reciprocal, when they may be so compar'd, that the antecedent of one proportion and the consequent of another are both found in the same figure. 'That is, when the Analogy begins and ends 'in the same figure. As if AB has the same pro- 'portion to CD, as DE to BF.

3. A line is divided according to the extreme and middle proportion, when the whole line has the same proportion to the greater part, as the greater part to the less. 'As if 'AB has the same proportion

to

'to AC, as AC to CB: the line AB is divided 'according to the extreme and middle propor- 'tion.



4. The height of any figure is a perpendicular drawn from its summity to its base. 'As in the triangles ABC, 'EFG, the perpendiculars 'EH and AD, whether they 'fall within or without the triangles, are their 'heights. Hence it follows, that all triangles 'and parallelograms, that have equal heights, 'may be plac'd within the same parallels. For 'having set their bases upon the line HC, if the 'perpendiculars DA and HE be equal, the lines 'EA and HC will be parallels.

5. A Proportion is said to be compounded of many others, when the *quantities of those proportions being multiply'd, make another. "To understand the true intent of 'this definition, it must be observ'd, that every 'Proportion, at least, every *rational Proportion, takes its De'nomination from a certain num'ber, denoting that respect or relation that the 'antecedent of the proportion bears to the con'sequent. As if two magnitudes were propos'd, 'one of twelve foot in length, and the other of 'six; we should call that proportion of 12 to

* *Denominators.*

* *Expressible by true numbers.*

'6 the double proportion. In like manner if '4 and 12 were propos'd, we ſhould give that 'the name of ſubtriple proportion, $\frac{1}{3}$ being its 'Denominator; importing as much as that the 'proportion of 4 to 12, is the ſame as that of '$\frac{1}{3}$ to unity, or as one to three. This Denomi-'nator is call'd the *quantity* of the Proportion. 'Suppoſe therefore three terms were given, 12, '6, and 2; the firſt proportion of 12 to 6 being 'double, its Denominator is two; the ſecond of '6 to 2 being triple, its Denominator is three; 'the proportion therefore of 12 to 2 is ſaid to 'be compounded of that of 12 to 6, and of 6 'to 2, the double and the triple proportion. 'To find therefore the Denominator of the pro-'portion of 12 to 2, multiply three by two, and 'the product ſix will ſhow the proportion of '12 to 2 to be ſextuple, *i.e.* as one to ſix. This 'is that which *Mathematicians* commonly un-'derſtand by compounded Proportion, though 'methinks it might more properly have been 'call'd Proportion multiply'd.

PROP.

PROPOSITION I.

A THEOREM.

Parallelograms, and triangles, of the same hight, are in the same proportion as their bases.

SUppose the triangles AGC, and DEM, to have the same hight; and to be plac'd between the same parallels, AD, and GM: I say, they will have the same proportion as their bases GC, and EM. Divide the base EM into as many equal parts as you please, and draw lines from the point D to each division, as DF, DH, DL. In like manner divide the line CG into parts equal to those of the line EM, and draw lines from its summity A to those divisions, as AB, AI, *&c.* All those little triangles, being enclos'd within the same parallels, and having equal bases, are equal, (*by the* 38. 1.)

Demonstration.

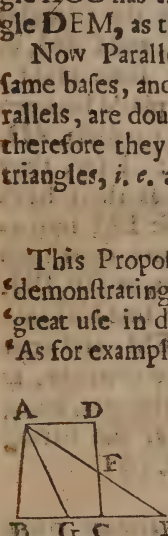
The base GC contains so many aliquot parts of EM, as there are parts found in it equal to EF; but as many parts equal to EF as are found in the base GC, so many little triangles are contain'd in the triangle AGC, equal to those

those contain'd in DEM; which being equal among themselves are the aliquot parts of the triangle DEM. As oft therefore as the base GC contains those aliquot parts of EM, so oft does the triangle AGC contain the aliquot parts of the triangle DEM; which also will happen in every division whatsoever: therefore the triangle AGC has the same proportion to the triangle DEM, as the base GC to the base EM.

Now Parallelograms, describ'd upon the same bases, and enclos'd between the same parallels, are double the triangles, (*by the* 41.1.) therefore they are in the same proportion as the triangles, *i. e.* as their bases.

The USE.

This Proposition is not only serviceable in 'demonstrating those that follow, but also of 'great use in dividing large fields, or plains. 'As for example, suppose you were to take the 'third part of the *Trapesium* 'ABCD, having the sides AD 'and BC parallel; produce the 'line BC to E, so that CE 'may be equal to AD; and 'and taking BG the third part 'of BE, draw AG: I say the tri-'angle ABG is the third part of the *Trapesium* 'ABCD.



Demon-

Demonstration.

'The triangles ADF, and FCE, are equian-'gular, because the lines AD and CE are pa-'rallels, and their sides AD and CE are equal: 'therefore (*by the* 26. 1.) the triangles are e-'qual, and consequently the triangle ABE is 'equal to the *trapesium* ABCD. But the tri-'angle ABG is the third part of the triangle 'ABE, (*by the preceding*,) therefore the trian-'gle ABG is the third part of the *trapesium* 'ABCD,

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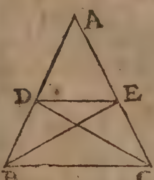
PROPOSITION II.

A THEOREM.

A line drawn in a triangle parallel to its base divides its sides proportionally; and the line that divides the sides of a triangle proportionally, will be parallel to its base.



IF in the triangle ABC the line DE be parallel to the base BC, the sides AB and AC will be divided proportionally, *i. e.* AD will have the same proportion to DB, as AE to EC. Draw the lines DC and BE. The triangles DBE, and DEC, having the same base

base DE, and being enclos'd within the same parallels DE and BC, are equal, (*by the* 37.1.)

Demonstration.

The triangles ADE and DBE have the same hight E, taking AD and DB for their bases, and therefore may be plac'd within the line AB and another parallel to it drawn through the point E, and consequently have the same proportion as their bases, (*by the* 1.) *i.e.* the triangle ADE has the same proportion to the triangle DBE, or its equal CED, as AD to DB. But the triangle ADE has likewise the same proportion
EC to the triangle CED, as AE to AC; and therefore AD has the same proportion to DB, as AE to EC.

Secondly, suppose AE to have the same proportion to EC, as AD to AB: I say the lines DE and BC will be parallels.

Demonstration.

AD has the same proportion to DB, as the triangle ADE to the triangle DEB, (*by the* 1.) and AE has the same proportion to EC, as the triangle ADE to the triangle CED; and consequently the triangle ADE has the same proportion to the triangle DEB, as the same ADE to the triangle CED. Therefore (*by the* 9.5.) the triangles BDE, and CED, are equal, and (*by the* 39 1.) between the same parallels. Therefore the lines DE and BC are parallels.

The

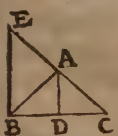
The USE.

'This Propoſition is abſolutely neceſſary for 'the demonſtration of thoſe that follow. It may 'alſo be ſerviceable in taking Dimenſions. As, 'in the following figure, if you were to mea-'ſure the hight of BE ; having erected a ſtaff or 'pole DA, there will be the ſame proportion of 'CD to DA, as of BC to BE.

PROPOSITION III.

A THEOREM.

A line that divides an angle of a triangle into two equal parts, divides its baſe into two parts which have the ſame proportion as the ſides. And if it divide the baſe into two parts proportional to the ſides, it will divide the angle into two equal parts.



IF the line AD divide the angle BAC into two equal parts, AB will have the ſame proportion to AC as BD to DC. Produce the ſide CA, and take AE equal to AB ; then draw the line EB.

Demonſtration.

The external angle CAB is equal to the two internal angles AEB, and ABE: which being equal, (*by the* 5. 1.) becauſe the ſides AE and

AB are equal; the angle BAD, which is the half of BAC, will be equal to one of them, suppose ABE; therefore (*by the* 27. 1.) the lines AD and EB are parallel, and (*by the* 2.) there is the same proportion of EA or AB to AC, as of BD to DC.

Secondly, if AB has the same proportion to AC as BD to DC, the angle BAC will be divided into two equal parts.

Demonstration.

AB or EA has the same proportion to AC; as BD to DC: therefore EB and AD are parallel; and (*by the* 29.1.) the alternate angles EBA and BAD, the internal BEA, and the external DAC, will be equal: and the angles BEA and EBA being equal, the angles BAD and DAC will be also equal. Therefore the angle BAC will be divided into two equal parts.

The USE.

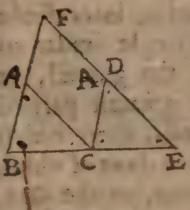
'We make use of this Proposition chiefly to 'find the proportion of the sides of a triangle.

PROP.

PROPOSITION IV.

A THEOREM.

The sides of equiangular triangles are proportional.



IF the triangles ABC, and DCE, be equiangular, *i. e.* if the angles ABC and DCE, BAC and CDE be equal, AB will have the same proportion to BC as DC to CE, and also AC will have the same proportion to CB as DE to EC. Joyn the triangles after such a manner, that the bases BC and CE may be upon the same line, and produce the sides BA and ED till they meet in F; since the angles ACB and DEC are equal, the lines AC and FE are parallels, [*by the* 28. 1.] and by the same reason CD and BF are parallels, and therefore AFDC a Parallelogram.

Demonstration.

In the triangle BFE, AC is parallel to FE, therefore [*by the* 2.] AB has the same proportion to AF, or, which is equal to it, CD, as BC to CE: and alternatively, AB has the same proportion to BC, as CD to CE. In like manner in the same triangle, CD being parallel to

BF, FD or AC has the ſame proportion to DE, as BC to CE, [*by the* 2.] and alternatively, AC has the ſame proportion to BC, as DE to CE.

The USE.

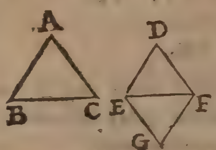
'This Propoſition is of ſo general uſe, that it 'may paſs for a moſt univerſal principle in tak-'ing all manner of Dimenſions. For in the firſt 'place all the methods of meaſuring inacceſſible 'lines, by deſcribing a ſmall triangle ſimilar to 'that which is form'd upon the ground, is 'founded upon it; as alſo the greateſt part of 'thoſe *Mathematical Inſtruments*, upon which 'are deſcrib'd triangles, ſimilar to thoſe of which 'we deſire to take the dimenſions, as the *Geo-'metrical ſquare*, the *Pantometer*, the *Arbaleſt* 'or *Croſs-ſtaff*, &c. Nor could we know how to 'raiſe the Plane of any place, but by the help 'thereof. So that in fine, to unfold all the uſes 'of this Propoſition, it would be neceſſary to 'tranſcribe the whole firſt book of *Practical Ge-'ometry*.

PROP.

PROPOSITION V.

A THEOREM.

Triangles, which have proportional ſides, are equiangular.



IF the triangles ABC and DEF have proportional ſides, *i,e.* if AB has the ſame proportion to BC as DE to *EF*, and alſo AB the ſame proportion to AC as DE to *DF* ; the angles ABC and DE*F*, A and D, C and *F*, will be equal. Make the angle *FEG* equal to the angle B, and *EFG* equal to the angle C.

Demonſtration.

The triangles ABC and EFG have two angles equal, therefore (*by Coroll.* 2. *of the* 32. 1.) they are equiangular ; and (*by the* 4.) AB has the ſame proportion to BC as GE to EF. Now tis ſuppos'd, that DE has the ſame proportion to EF as AB to BC, therefore DE has the ſame proportion to EF as EG to EF ; and conſequently (*by the* 9. 5.) DE and EG are equal. After the ſame manner DF may be prov'd equal to FG, and conſequently (*by the* 8. 1.) the triangles DEF and GEF are equiangular. But the angle

angle GEF was made equal to B, therefore the angle DEF is equal to the angle B, and the angle DFE to the angle C; and consequently, the triangles ABC and DEF are equiangular.

PROPOSITION VI.

A THEOREM.

Triangles, which have each one of their angles equal to one of the other, and the sides containing that angle proportional, are equiangular.

If the triangles ABC and DEF (*see fig. preced.*) have the angles B and E equal; and the side AB has the same proportion to BC as DE to *EF*, the triangles ABC and DE*F* will be equiangular. Make the angle FEG equal to the angle B, and the angle EFG equal to the angle C.

Demonstration.

The triangles ABC and GE*F* are equiangular, (*by Coroll.* 2. *of the* 32. 1.) therefore AB has the same proportion to BC as GE to *EF*, (*by the* 4.) But as AB to BC, so is DE to *EF*; therefore DE has the same proportion to *EF* as GE to the same *EF*; and therefore (*by the* 9. 5.) DE and EG are equal; and the triangles DE*F* and GFE, having the angles DE*F* and GE*F*, each equal to the angle B, and the sides

DE

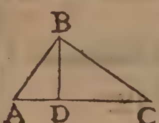
DE and EG equal, with the ſide *EF* common to both, will be equal in all reſpects, (*by the* 4. 1.) therefore they will be equiangular; and the triangle EG*F* being equiangular to ABC, the triangles ABC and DE*F* are equiangular.

The Seventh Propoſition is of no uſe.

PROPOSITION VIII.

A THEOREM.

A perpendicular, drawn from the right angle of a rectangular triangle to the oppoſite ſide, divides the triangle into two others ſimilar to it,



IF the perpendicular BD be drawn from the right angle ABC to the oppoſite ſide AC, it will divide the rectangular triangle ABC into two triangles ADB and BDC, which will be ſimilar, or equiangular to the triangle ABC.

Demonſtration.

The triangles ABC and ADB have the ſame angle A, and the angles ABC and ADB right angles, therefore they are equiangular, [*by Coroll.* 2. *of the* 32. 1.] In like manner the triangles BDC and ABC have the angle C common to both, and the angles ABC and BDC right angles; therefore they alſo are equiangular. There-

Therefore the triangles ABC, BDC, and ADB, are similar triangles.

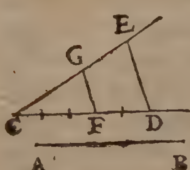
The USE.

'By the help of this Proposition inaccessible 'distances may be measur'd by a Carpenter's 'square. As for example, if I were to measure 'the distance DC; having drawn the perpen-'dicular BD, and plac'd my square upon the 'point B in such a manner, that by one of its 'sides BC I could observe the point C, and by 'the other the point A: 'tis evident, there 'would be the same proportion of AD to DB, 'as of DB to DC. Therefore multiplying DB 'by its self, and dividing the product by AD, 'the Quotient would be DC.

PROPOSITION IX.

A PROBLEM.

Of a line given to cut off what part you please.



LET the line propos'd be AB, from which you desire to take away three fifth parts. Make an angle ECD, and upon one of its sides CD take five equal parts, three of which shall be contain'd in CF:

CF: then taking CE equal to AB, draw the line DE, and another parallel to that FG: the line CG will contain three fifth parts of CE or AB.

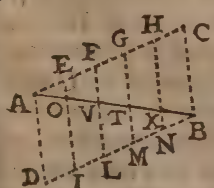
Demonstration.

In the triangle ECD, FG being parallel to the base DE, CF will have the same proportion to FD as CG to GE, [*by the* 2.] and compounding them, [*by the* 18. 5.] CE will have the same proportion to CG as CD to CF; and (*by the Coroll, of the* 16. 5.) CG will have the same proportion to CE as CF to CD. But CF contains three fifths of CD, therefore CG will contain three fifths of CE or AB.

PROPOSITION X.

A PROBLEM.

To divide a line after the same manner as another line given is divided.



IF you would divide the line AB after the same manner as AC is divided, make with the two lines the angle CAB of what magnitude you please; then draw the line BC, and parallel to

it the lines EO, FV, and the reſt. The line AB will be divided after the ſame manner that AC is.

Demonſtration.

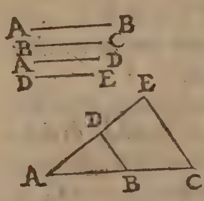
Since in the triangle BAC, HX is drawn parallel to the baſe BC, it will divide the ſides AB and AC proportionally, (*by the* 2.) and the ſame may be ſaid of all the reſt.

To do this more eaſily, you may draw the line BD parallel to AC; and transferring the diviſions of AC to BD, draw lines from one to the other.

PROPOSITION XI.

A PROBLEM.

Two lines being given to find a third proportional



IF you would find a third proportional to the lines AB and BC, *i. e.* that there may be the ſame proportion of AB to BC, as of BC to the line ſought; make at pleaſure the angle EAC, and upon one of its ſides take the lines AB and BC, one immediately after the other; and upon the other ſide take AD equal to BC: then draw the line BD, and parallel to it the line CE; and the line DE will be that which you ſeek.

De-

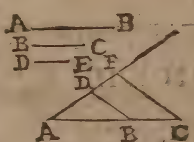
Demonstration.

In the triangle EAC the line DB is parallel to the base CE, therefore [*by the* 2.] there is the same proportion of AB to BC, as of AD or BC to DE.

PROPOSITION XII.

A PROBLEM.

Three lines being given to find a fourth proportional.



LET the three lines propos'd, to which you are to find a fourth proportional, be AB, BC, and DE. Make at pleasure the angle FAC, and take upon AC the lines AB and BC, and upon AF the line AD equal to DE; then draw the line DB, and parallel to it FC: I say, that DF is the line sought, *i. e.* there is the same proportion of AB to BC as of DE or AD to DF.

Demonstration.

In the triangle FAC the line DB is parallel to the base FC: there is therefore the same proportion of AB to BC, as of AD to DF, [*by the* 2.]

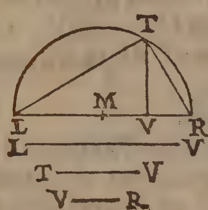
The U S E.

'The uſe of the *Compaſs* of *Proportion* is 'grounded upon theſe four Propoſitions; for 'that Inſtrument teaching us to divide a line as 'we pleaſe; to make uſe of the R*ule of* T*hree*, 'without the help of *Arithmetick*; to extract 'the Square, and Cubick roots; to double the 'Cube; to meaſure all ſorts of Triangles; to 'find the capacity of Superficies's, and the ſoli-'dity of bodies; and to augment or diminiſh 'any figure, according to what proportion we 'deſire; all theſe operations are demonſtrated 'by the preceding Propoſitions.

PROPOSITION XIII.

A PROBLEM.

Two lines being given to aſſign a middle proportional.



IF you deſire a middle proportional between the lines LV, and VR; having plac'd them ſo, that they make but one right line LR, divide that line into two equal parts in M; and having

ing describ'd a semicircle LTR from the center M, draw the perpendicular VT, which will be a middle proportional between LV and VR. Draw the lines LT, and TR.

Demonstration.

The angle LTR, being describ'd in a semicircle, is a right angle, (*by the* 31. 3.) and (*by the* 8.) the triangles LVT and TVR are similar; therefore there is the same proportion of LV to VT in the triangle LVT, as of VT to VR in the triangle TVR, (*by the* 4.) therefore VT is a middle proportional between LV and VR.

The USE.

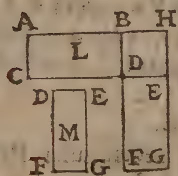
'By this Proposition any rectangular Parallelogram may be reduc'd to a Square. For example, in the Rectangle contain'd under LV and VR, the square of VT is equal to the Rectangle under LV and VR; as I shall hereafter demonstrate.

PROP.

PROPOSITION XIV.

A Theorem.

Equal equiangular Parallelograms have their sides reciprocal; and equiangular Parallelograms, that have their sides reciprocal, are equal.



IF the parallelograms L and M be equiangular and equal, their sides will be reciprocal, *i. e.* CD will have the same proportion to DE as FD to DB. Since they have equal angles, they may be so joyn'd together, that the sides CD and DE, FD and DB, will concur in two right lines, (*by the coroll. of the* 15. 1.) producing therefore the sides AB and GE, you compleat the parallelogram BDEH.

Demonstration.

The parallelograms L and M being equal, will have the same proportion to the parallelogram BDEH. But the proportion of L to BDEH, is as the base CD to the base DE; and that of M, or DFGE, to BDEH, is as the base FD to the base DB, (*by the* 1.) Therefore CD has the same proportion to DE, as FD to DB.

Se-

Secondly, if the Parallelograms L and M be equiangular, and have their ſides reciprocal, they will be equal.

Demonſtration.

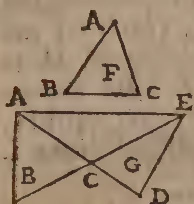
The ſides of the parallelograms are ſuppos'd to be reciprocal, *i. e.* that there is the ſame proportion of CD to DE, as of FD to DB: but as the baſe CD is to DE, ſo is the parallelogram L to the parallelogram BDEH, (*by the* 1.) and as FD to DB, ſo is the parallelogram M to BDEH; therefore L has the ſame proportion to BDEH, as M to the ſame BDEH; therefore (*by the* 9. 5.) the parallelograms L and M are equal.

PROP.

PROPOSITION XV.

A THEOREM.

Equal triangles, that have one angle equal each to the other, have the ſides, that form that angle, reciprocal: and if thoſe ſides be reciprocal, they will be equal,



IF the triangles F and G, being equal, have the angles ACB and DCE equal, their ſides that form thoſe angles will be reciprocal, *i. e.* BC will have the ſame proportion to CE as CD to CA. Place the triangles ſo, that the ſides CD and CA may make one right line; and then becauſe the angles ACB and DCE are ſuppos'd to be equal, BC and CE will alſo make one right line, (*by Coroll. of the* 15. 1.) Draw the line AE.

Demonſtration.

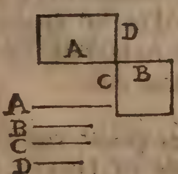
The triangle ABC has the ſame proportion to the triangle ACE, as the triangle ECD, equal to the former, to the ſame ACE, (*by the* 7. 5.) But as ABC to ACE, ſo is the baſe BC to the baſe CE, [*by the* 1.] having both the ſame hight A; and as ECD to ACE, ſo is the baſe CD

CD to CA, (*by the same:*) therefore BC has the same proportion to CE, as CD to CA. But if the sides be suppos'd reciprocal, *i. e.* that BC has the same proportion to CE as CD to CA, the triangles ABC and CDE will be equal, because they will both have the same proportion to ACE.

PROPOSITION XVI.

A Theorem.

If four lines be proportional, the rectangle contain'd under the first and the fourth, will be equal to the rectangle contain'd under the second and the third. And if the rectangle contain'd under the extremes be equal to that contain'd under the middle termes, the four lines will be proportional.



If the lines A, B,C,D, be proportional, *i. e.* if as A to B, so C to D, the rectangle contain'd under the first A, and the fourth D, will be equal to the rectangle contain'd under B and C.

Demonstration.

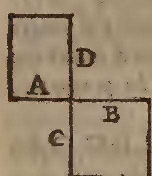
The rectangles have one angle equal each to th' other, because 'tis a right angle in both; their sides also are reciprocal: therefore they are equal, [*by the* 14.]

In like manner, if they are equal, their ſides will be reciprocal, *i.e.* A will have the ſame proportion to B as C to D.

PROPOSITION XVII.

A Theorem.

If three lines be proportional, the rectangle contain'd under the firſt and the third, will be equal to the ſquare of the middle terme. And if the ſquare of the middle terme be equal to the rectangle under the extremes, the three lines are proportional.



IF the three lines A,B,D, be proportional, the rectangle contain'd under A and D will be equal to the ſquare of B. Take C equal to B, and there will be the ſame proportion of A to B as of C to D; therefore the four lines are proportional.

Demonſtration.

The rectangle under A and D will be equal to that under B and C, (*by the preceding*,) but the laſt rectangle is a ſquare, the lines B and C being equal: therefore the rectangle contain'd under A and D is equal to the ſquare of B.

In like manner, if the rectangle under A and D

D be equal to the ſquare of B, A will have the ſame proportion to B as C to D: and B and C being equal, A will have the ſame proportion to B, as B to D.

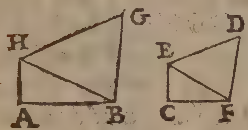
The U S E.

'By theſe four propoſitions may be demon-'ſtrated that Rule in *Arithmetick*, which is 'commonly call'd the *Rule of* T*hree*; and con-'ſequently, the Rules of *Fellowſhip*, of *Falſe*, and 'all thoſe others that depend upon Proportion. 'For example, ſuppoſe Three numbers given, 'A 8, B 6, and C 4, and it be requir'd to find 'a fourth proportional number; which taking 'as found, I will call D. The rectangle then 'contain'd under A and D, is equal to that un-'der B and C. But I may have this latter rect-'angle by multiplying B by C, *i. e.* ſix by four, 'the product will be twenty four; therefore the 'rectangle contain'd under A and D is alſo 'twenty four; and therefore dividing that num-'ber by A, which is 8, the Quotient three will 'be the number ſought.

PROPOSITION XVIII.

A PROBLEM.

To describe a Polygon similar to another upon a line given.



LET AB be the line assign'd, upon which you are requir'd to describe a Polygon similar to the Polygon CFDE; and having divided the polygon CFDE into triangles, upon the line AB make a triangle ABH similar to the triangle CFE, *i. e.* make the angle ABH equal to the angle CFE, and BAH equal to FCE, for then the triangles ABH and CFE will be equiangular, [*by Corol.*2. *of the* 32.1.] Make also upon the line BH a triangle equiangular to FDE.

Demonstration.

Since the triangles, which are part of the polygons, are equiangular, the two polygons are equiangular. Further, since the triangles ABH and CFE are equiangular, AB will have the same proportion to BH as CF to FE, [*by the* 4.] In like manner, the triangles HBG and EFD being equiangular, BH will have the same proportion to BG as FE to FD: and by equality, (accord-

[*according to defin.* 18. 5.] AB will have the same proportion to EG, as CF to FD. And the same may be said of all the other sides. Therefore (*by defin.*1.) the polygons are similar.

The USE.

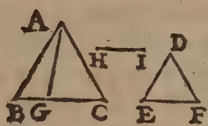
'Upon this proposition is grounded the greatest part of *Practical Geometry*, that relates to 'the raising the plane of any place, as of a 'building, field, forrest, or a whole country. For 'having divided a line into equal parts, to answer the feet or yards contain'd in the plane, 'you may describe a figure similar to, but less 'than, the Original, in which you may see the 'proportion of all its lines. And having by experience found it much more easie to travel 'upon paper, than to take a tedious journey either by land or water, this proposition will 'likewise afford us assistance in this respect, informing us in almost all the parts of *Geodesia*, 'and *Chorography*; and giving instructions how 'to compose *Geographical Charts*, and *Maps*; 'which are nothing else but methods of reduceing great figures to small. Further, the use of 'this proposition extends it self to almost all 'those Arts, that require the design and model 'of their works before hand.

PROP.

PROPOSITION XIX.

A THEOREM.

Similar triangles are in the duplicate proportion of their homologous sides.



IF the triangles ABC and DEF be similar or equiangular, they will be in the duplicate proportion of their homologous sides BC, EF, *i. e.* the proportion of the triangle ABC to the triangle DEF will be the duplicate of the proportion of BC to EF; so that finding a third proportional HI to the lines BC and EF, and making BC to have the same proportion to EF as EF to HI, the triangle ABC will have the same proportion to DEF as the line BC to the line HI; which is to have to it a duplicate proportion, [*by defin.* 11.5.] Take BG equal to to HI, and draw the line AG,

Demonstration.

The angles B and E of the triangles ABG and DEF are equal; and besides, since the triangles ABC and DEF are similar, AB will have the same proportion to DE as BC to EF, [*by the fourth.*] But as BC to EF, so EF to HI or BG; therefore as AB to DE, so *EF* to BG; and

and consequently, the sides of the triangles ABG and DEF being reciprocal, the triangles will be equal, [*by the* 15.] And [*by the* 1.] the triangle ABC has the same proportion to the triangle ABG, as BC to BG or HI: therefore the triangle ABC has the same proportion to the triangle DEF, as BC to HI.

The USE.

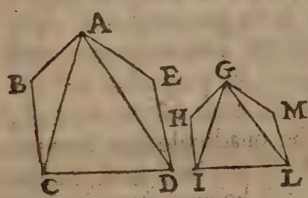
These Propositions may help to correct the 'error of those, who are apt to imagine similar 'figures to have the same proportion as their 'sides. For if two squares, two pentagons, two 'hexagons, or two circles, be propos'd, and the 'side of the first be double that of the second, 'the first figure will be quadruple the second: if the side of the first be triple that of the second, the first figure will be nine times greater than the second. Therefore to make a square triple to another, you must seek a middle proportional between one and three, and you'l find for the side of your triple figure almost $1\frac{3}{4}$.

PROP.

PROPOSITION XX.

A Theorem.

Similar Polygons may be divided into an equal number of triangles, and are in the duplicate proportion of their homologous sides.



If the Polygons ABCDE and GHILM be similar, they may be divided into an equal number of similar triangles, which will be the similar parts of their wholes. Draw the lines AC, AD, GI, GL.

Demonstration.

Since the Polygons are similar, their angles *B* and H will be equal; and *AB* will have the same proportion to *BC* as GH to HI, (*by defin.* 1.) therefore the triangles ABC and GHI are similar, (*by the* 6.) and (*by the* 4.) *BC* has the same proportion to CA as HI to GI. Further, because CD has the same proportion to *BC* as IL to IH, and *BC* the same to *CA* as HI to IG; by equality, *CD* will have the same proportion to *CA* as IL to GI. Now the angles *BCD*

BCD and HIL being equal, if the angles *ACB* and GIH, which are equal, be taken from them, the angles *ACD* and GIL will remain equal. Therefore (*by the 6.*) the triangles ACD and GIL will be ſimilar. In like manner, tis eaſie to run over all the triangles of the Polygons, and to prove them ſimilar.

I add further, that the triangles are in the ſame proportion as the polygons.

Demonſtration.

Since all the triangles are ſimilar, their ſides will be proportional, (*by the* 4.) but each triangle to its ſimilar is in the duplicate proportion of the homologous ſides, (*by the* 19.) therefore every triangle of one polygon to every triangle of the other is in the duplicate proportion of its ſides; which being the ſame, the duplicate proportion muſt be the ſame; and there will be the ſame proportion of each triangle to its ſimilar, as of all the triangles of one polygon to all the triangles of the other polygon, (*by the* 12. 5.) *i.e.* of one polygon to the other.

Coroll. 1. Similar polygons are in the duplicate proportion of their ſides.

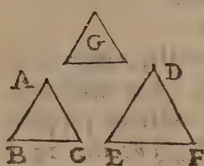
Coroll. 2. If three lines be in continual proportion, a polygon deſcrib'd upon the firſt will have the ſame proportion to a polygon deſcrib'd upon the ſecond, as the firſt line to the third, *i.e.* it will be in the duplicate propor-

tion of that of the firſt line to the ſecond.

PROPOSITION XXI.

A Theorem.

Polygons, that are ſimilar to another polygon, are ſo alſo amongſt themſelves.



IF two polygons be ſimilar to a third, they will be ſo alſo betwixt themſelves. For they may each be divided into as many ſimilar triangles, as are in the third. But triangles ſimilar to a third, are alſo ſimilar amongſt themſelves; becauſe angles equal to a third, are equal amongſt themſelves; and the angles of the triangles being equal, thoſe of the polygons being compounded of them muſt be ſo likewiſe.

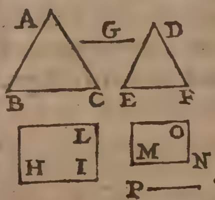
I add, that the ſides of the triangles being proportional, thoſe of the polygons muſt be ſo alſo, becauſe they are the ſame.

PROP.

PROPOSITION XXII.

A Theorem.

Similar polygons describ'd upon four proportional lines, are also proportional. And if the polygons be proportional, the lines are so too.



IF BC has the same proportion to EF as HI to MN, the polygon ABC will also have the same proportion to the similar polygon DEF, as HL to its similar polygon MO. Seek a third proportional G to the lines BC and EF, and to the lines HI and MO another third proportional P, (*by the* 11.) Since BC has the same proportion to EF as HI to MN, and EF to G as MN to *P*; by equality, BC will have the same proportion to G, as HI to *P*: and this proportion will be the double of that of BC to EF, or HI to MN.

Demonstration.

The polygon ABC to the polygon DFF is in the duplicate proportion of that of BC to EF, (*by the* 20.) that is, as *BC* to G; and the polygon HL has the same proportion to MO, as HI to *P*. Therefore ABC has the same pro-

portion to DEF, as HL to MO.

And if the ſimilar polygons be proportional, the lines being in the ſubduplicate proportion to them, will be alſo proportional.

The USE.

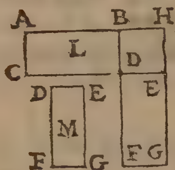
A,	B ;	C,	D,
3,	2 ;	6,	4,
9,	4 ;	36,	16,
E,	F ;	G,	H,

This propoſition may be eaſily apply'd to numbers, if the 'numbers A,B,C,D, be propor-'tional, their ſquares E,F,G,H, 'will be ſo alſo; which is very 'ſerviceable in *Arithmetick*, and more in *Alge-'bra.*

PROPOSITION XXIII.

A THEOREM.

Equiangular Parallelograms are in the proportion compounded of the proportions of their ſides.



IF the parallelograms L and M be equiangular, the proportion of L to M will be compounded of that of AB to DE, and that of BD to DF. Joyn the parallelograms ſo, that their ſides BD and DF may make but one right line, as alſo CD and DE another; which, the paral-

rallelograms being equiangular, may be done; [*by the coroll. of the* 15. 1.] and compleat the parallelogram BDEH.

Demonſtration.

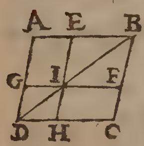
The parallelogram L has the ſame proportion to the parallelogram BDEH, as the baſe AB to the baſe BH or DE, [*by the* 1.] and the parallelogram BDEH has the ſame proportion to the parallelogram DFGE, *i.e.* M, as the baſe BD to DF. But the proportion of the parallelogram L to the parallelogram M is compounded of that of L to the parallelogram BDEH, and of that of BDEH to the parallelogram M. Therefore the proportion of L to M is compounded of that of *AB* to DE, and that of BD to *DF*. For example, let *AB* be 8, DE 5, *BD* 4, *DF* 7; and make as 4 to 7, ſo 5 to $8\frac{3}{4}$; by which means you will have three numbers, 8, 5, and $8\frac{3}{4}$; 8 to 5 being the proportion of the parallelogram L to BDEH, which is that of AB to DE; and 5 to $8\frac{3}{4}$ that of the parallelogram BDEH to M. Taking away therefore the middle term five, there will remain 8 to $8\frac{3}{4}$ for the proportion compounded of the two.

PROP.

PROPOSITION XXIV.

A THEOREM.

In all parallelograms, those through which the diameter passes, are similar to the great one.



SUppose the diameter of the parallelogram AC pass thro the parallelograms *EF*, GH: I say they are similar to the parallelogram AC.

Demonstration.

The parallelograms AC and *EF* have the same angle B: and because in the triangle BCD, *IF* is parallel to the base DC, the triangles BFI and BCD are equiangular. Therefore [*by the fourth*] *BC* has the same proportion to CD as *BF* to *FI*, and consequently the sides are in the same proportion. In like manner IH being parallel to BC, DH will have the same proportion to HI as DC to BC; the angles are also equal, all the sides being parallels: therefore [*by defin.*1.] the parallelograms *EF* and GH are similar to the parallelogram AC.

The USE.

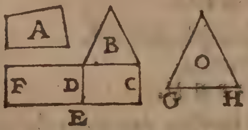
I have made use of this proposition to demonstrate the 10. proposition of my last book of

'of *Perspectives*, where I have shown a way to 'draw an Image similar to the Original, by a pa-'rallelogram compos'd of four Rules.

PROPOSITION XXV.

A PROBLEM.

To describe a polygon similar to one polygon given and equal to another.



If you desire to describe a polygon equal to the rectilineal A, and similar to the polygon B, make a parallelogram CE equal to the polygon B, [*by the* 44.1.] and upon the line DE make another parallelogram equal to the rectilineal A, [*by the* 45.1.] Then find a middle proportional GH between CD and *DF*, [*by the* 13.] Lastly, make upon GH a polygon O, similar to B, [*by the* 18.] which will be equal to the rectilineal A.

Demonstration.

Since CD, GH, and *DF*, are in a continual proportion, the rectilineal *B* describ'd upon the first, will have the same proportion to the rectilineal O describ'd upon the second, as CD to *DF*, [*by coroll.* 2. *of the* 20.] But as CD to *DF*, so is the parallelogram CE to *FE*, or B to A, which

which are equal to them. Therefore B has the same proportion to O as B to A, and consequently [*by the* 9.5.] A and O are equal.

The USE.

'This proposition teaches how to change one 'figure for another, retaining still its equality 'to a third; which is very useful in *practical* '*Geometry*, for the reducing all figures to 'squares.

PROPOSITION XXVI.

A THEOREM.

If in one angle of a parallelogram you describe a less, similar to the former, the diameter of the greater will fall upon the angle of the less.

IF in the angle D of the parallelogram AC you describe a lesser parallelogram DG, similar to the other, the diameter BG will pass by the point G. For if it do not pass by that point, suppose it then to pass by the point I, and to make the line BID. Draw the line IE parallel to HD.

Demonstration.

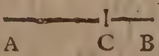
The parallelogram DI would be similar to the

the parallelogram AC, (*by the* 24.) But the parallelogram DG is alſo ſuppos'd ſimilar to it, therefore the parallelograms DI and DG would be ſimilar, which is impoſſible; for ſo HI would have the ſame proportion to IE or GF, as HG to the ſame GF; and (*by the* 9. 5.) the lines HI and HG would be equal.

PROPOSITION XXX.

A Problem.

To divide a line according to the extreme and middle proportion.



LET AB be the line propos'd to be divided according to the extreme and middle proportion, *i. e.* ſo, that AB may have the ſame proportion to AC as AC to CB. Divide the line AB (*by the* 11. 2.) ſo, that the rectangle contain'd under AB and CB may be equal to the ſquare of AC.

Demonſtration.

Since the rectangle under AB and CB is equal to the ſquare of AC, AB will have the ſame proportion to AC as AC to CB, (*by the* 17.)

The USE.

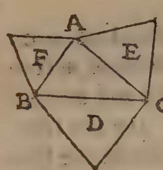
'This propoſition is neceſſary in the Thir'teenth book of *Euclid*, for the finding the ſide

'of five regular bodies. And Frier *Lucas*, of the *Holy Sepulcher*, has compos'd a whole book 'concerning the properties of a line divided 'according to the extreme and middle proportion.

PROPOSITION XXXI.

A THEOREM.

A polygon describ'd upon the base of a rectangular triangle, is equal to the two similar polygons describ'd upon the other sides of the same triangle.



IF the angle BAC of the triangle ABC be a right angle, the polygon D, describ'd upon its base BC, will be equal to the two similar polygons F and E describ'd upon the sides AB and AC.

Demonstration.

The polygons D,E, and F, are amongst themselves in the duplicate proportion of their homologous sides BC, AC, and AB, (*by the* 20.) and if squares were describ'd upon the same sides, they also would amongst themselves be in the duplicate proportion of their sides; but [*by the* 47.1.] the square of BC would be equal to the

the ſquares of AC and AB : therefore the polygon D deſcrib'd upon the baſe BC, will be equal to the ſimilar polygons E, and F, deſcrib'd upon AB and AC.

The USE.

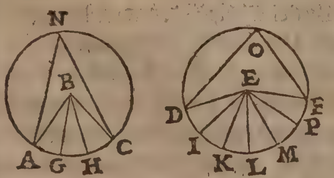
'This propoſition is made uſe of to augment 'or diminiſh all manner of figures, being more 'univerſal than the 47.1. which yet is exceed-'ing uſeful, in as much as almoſt all *Geometry* is 'grounded upon that principle.

The 32. Propoſition is uſeleſs.

PROPOSITION XXXIII.

A THEOREM.

In equal circles, the angles as well at the center as circumference, as alſo the ſector, are in the ſame proportion as the arches upon which they ſtand.



IF the circles ANC and DOF are equal, the angle ABC will have the the ſame proportion to the angle DEF as the arch AC to the arch DF. Suppoſe AG, GH, and HC, to be equal archs,

 and

and consequently the aliquot parts of AC ; and let DF be divided into as many parts, equal to AG, as it contains ; and draw the lines EI, EK, and the rest.

Demonstration.

All the angles, ABG, GBH, HBC, DEI, IEK, and the rest, are equal, (*by the* 27.3.) so that AG, an aliquot part of the arch AC, will be contain'd in the arch DF, as oft as the angle ABG, an aliquot part of the angle ABC, is contain'd in the angle DEF ; therefore the arch AC will have the same proportion to the arch DF, as the angle ABC to the angle DEF. And because N and O are the halves of the angles ABC and DEF, they will be in the same proportion as these : therefore the angle N has the same proportion to the angle O, as the arch AC to the arch DF.

The same holds likewise of the Sectors : for if you draw the lines AG, GH, HC, DI, IK, and the rest, they will be equal, (*by the* 29.3.) and each little sector will be divided into a triangle, and a segment. But the triangles will be equal, (*by the* 8.1.) and the little segments will also be equal, (*by the* 24.3.) therefore the whole little Sectors will be equal ; and consequently, as many aliquot parts of the arch AC as are contain'd in the arch DF, so many aliquot parts of the sector ABC will be contain'd in the sector DEF. Therefore the arch has the same proportion to the arch, as the sector to the sector.

THE

THE ELEVENTH BOOK OF THE ELEMENTS OF EUCLID.

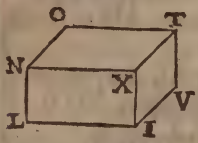
'THis Book eſtabliſhes the firſt principles re-
'lating to ſolid bodies ; inſomuch that it
'is impoſſible to know any thing certainly con-
'cerning the third *ſpecies* of quantity, without
'underſtanding what is herein taught. Upon
'which account the knowledge of it is abſolute-
'ly neceſſary to a through-inſight into the great-
'eſt part of *Mathematical* Treatiſes. In the firſt
'place, the Doctrine of the Sphere deliver'd by
'*Theodoſius* does ſuppoſe a perfect knowledge
'of the whole. In like manner *Spherical Trigo-
'nometry*, the third part of *Practical Geometry*,
'divers propoſitions of *Staticks* and *Geography*
'are built upon the principles of Solids. The
'main difficulties in *Gnomonicks*, *Conick Se-
'ctions*,

'*ctions*, and the Tracts concerning the cutting of 'pretious Stones, arising chiefly from their emi-'nencies and rais'd parts, not easily represented 'upon paper, and their being contain'd under 'many superficies's, are render'd intelligible 'and easie by the previous knowledge of the 'doctrine of Solids.

'I have omitted the seventh, eighth, ninth, 'and tenth Books of the *Elements* of *Euclid*, be-'ing of little or no use in any part of the *Ma-'thematicks*. And I have oft wondred how they 'obtein'd a place amongst the *Elements*, since tis 'evident *Euclid* compil'd them for no other end 'but to settle the doctrine of Incommensura-'bles; which being little better than a vain 'curiosity, ought not to be receiv'd into the 'books which treat of the *First Principles* of the 'Science, but to make a particular Treatise by it 'self. The same may be said of the Thirteenth 'book, and those that follow it. And therefore 'tis my opinion, that almost all parts of the *Ma-'thematicks* may sufficiently be understood by 'the help of these Eight books of the *Elements* 'of *Euclid*.

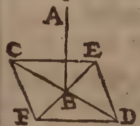
DE-

DEFINITIONS.

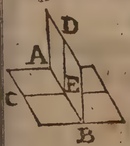


1. A Solid body is a quantity, that hath length, bredth, and depth,, or thickneſs. "As "the figure LT, whoſe "length is NX, bredth NO, and thickneſs "LN.

2. The extremes or terms of a ſolid body are ſuperficies's.



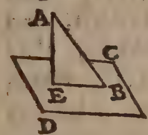
3. A Line is right, or perpendicular to a plane, when tis perpendicular to all the lines, which it meets in the plane. "As the "line AB will be right to the "plane CD, if it be perpendicular to the lines "CD and FE, which being drawn upon the "plane CD, paſs by the point B, ſo that the "angles ABC, ABD, ABE, and ABF, are right "angles.



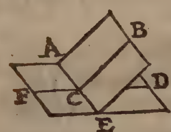
4. One plane is perpendicular to another, when a perpendicular line drawn upon one of them to the common ſection, is alſo perpendicular to the other.

" We call the line that is common

"mon to both the planes the common ſection "of the planes: As the line AB, which is as "well in the plane AC, as in the other AD. If "therefore the line DE, drawn on the plane "AD, perpendicular to AB, be alſo perpendi- "cular to the plane AC, the plane AD will be "right to the plane AC.



5. If the line AB be not perpendicular to the plane CD, and from the point A a perpendicular be drawn to it AE, and alſo the line BE; the angle ABE is the Inclination of the line AB to the plane CD.



6. The Inclination of one plane to another, is the acute angle form'd by the two perpendiculars drawn upon each plane to their common ſection. "As the Inclination of the plane AB to "the plane AD, is nothing elſe but the angle "BCD, form'd by the lines BC and CD, drawn "upon the two planes, perpendicular to their "common ſection AE.

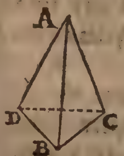
7. Planes are inclin'd after the ſame manner if their angles of Inclination be equal.

8. Planes are parallel, if being continu'd as far as you pleaſe, they ſtill retain the ſame diſtance one from the other.

9. Solid figures are ſimilar, which are contain'd within, or terminated by, an equal number

ber of ſimilar planes; as two Cubes. "This "definition does not agree to thoſe figures, "whoſe ſuperficies's are crooked; as the Sphere, "the Cylinder, and the Cone.

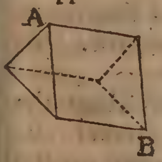
10. Equal and ſimilar ſolid figures are contain'd within, or terminated by, an equal number of equal and ſimilar planes. " Inſomuch, "that if they were ſuppos'd to penetrate each "other, neither of them would exceed, having "their ſides and angles equal.



11. A ſolid angle is the concourſe, or inclination, of divers lines, in different planes. " As the "concourſe of the lines AB, AC, "and AD, which are in different planes.

12. A Pyramid is a ſolid figure, terminated by triangles, whoſe baſes are in the ſame plane. "As the figure ABCD.

13. A Parallelepipedon is a ſolid figure contain'd within ſix quadrilateral planes, of which the oppoſites are parallel.



14. A Priſme is a ſolid figure, having two parallel planes ſimilar and equal, and the others Parallelograms. "As the "figure AB. Its oppoſite planes "may be polygons.

15. A Sphere is a ſolid figure, terminated by one only ſuperficies; from which divers

Q q lines

lines being drawn to a point in the middle of the figure, they will be all equal. "Some de- "fine a Sphere by the motion of a semicircle, "turn'd about upon its diameter, which re- "mains immoveable.

16. The Axis of a Sphere is that immove- able line about which the semicircle is turn'd.

17. The Center of the Sphere, is the same with that of the semicircle, by whose motion it is made.

18. The Diameter of a Sphere, is any line whatsoever passing through its center, and ter- minated at the superficies.

19. If a line, immoveable at one of its points, taken above the plane of a circle, be mov'd about the circumfe- rence, it will describe a Cone. "As if "the line AB, being fix'd at the point "A, be mov'd about the circumfe- "rence BED, it will describe the Cone ABED, "The point A will be its summity or vertex, "and the circle BED its base.

20. The Axis of a Cone, is the line drawn from the vertex to the center of the base. "As "AC.

21. If a line be mov'd about the circumference of two parallel cir- cles, so that it remains always parallel to a line drawn from the center of one of the circles to that of the o- ther,

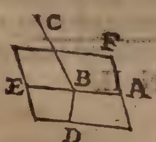
ther, *i. e.* the Axis, it will deſcribe a Cylinder.

22. Cones are ſaid to be right, when the axis is perpendicular to the plane of the baſe. Alſo right cones are ſimilar, when their axis's and the diameters of their baſes are in the ſame proportion. But Inclin'd cones are not ſimilar, unleſs they have a third condition; that their axis's be equally inclin'd to the planes of their baſes.

PROPOSITION I.

A THEOREM.

A right line cannot have one of its parts upon a plane, and the other above or below it.



IF the line AB be upon the plane AD, it will not, being continu'd, either riſe above or fall below it, but all its parts will lie upon the ſame. For if it be poſſible that BC can be a part of AB continued, draw upon the ſame plane AD the line BD perpendicular to AB, and alſo BE perpendicular to BD upon the ſame.

Demonſtration.

The angles ABD and DBE are two right angles; therefore (*by the* 14. 1.) AB and BE make but one right line, and conſequently BC

 is

is no part of the line AB continued : otherwise two right lines CB and EB would have the same part AB in common , which is repugnant to the 13. Axiom of the first book.

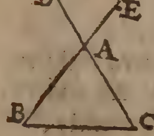
The USE.

Upon this proposition is built a principle in '*Gnomonicks*, by which we prove, that the *sha-* '*dow* of the *Style* cannot fall out of the plane of 'a great circle, in which is the Sun. For the 'extremity of the *Style* being taken for the cen- 'of the heavens, and consequently of all the 'greater circles, and the *shadow* being always in 'a right line of a Ray drawn from the Sun to the 'opacous body, and this ray being in the plane 'of this great circle, the shadow must be so 'likewise.

PROPOSITION II.

A THEOREM.

Lines that cut each other, are in the same plane, as are also all the parts of a triangle.



IF the two lines BE and CD cut each other at the point A; and a triangle be form'd by drawing the base BC; I say, all the parts of the triangle ABC are

are in the ſame plane, and alſo the lines BE and CD.

Demonſtration.

It cannot be ſaid that any part of the triangle ABC is in a plane, and another part of the ſame triangle not in the ſame, but it muſt be alſo affirm'd, that one part of a right line is in a plane, and another part of the ſame line is not in the ſame plane; which is contrary to *Prop.* 1. And becauſe the ſides of the triangle muſt be in the ſame plane in which is the triangle, the lines BE and CD will be alſo in the ſame plane.

The USE.

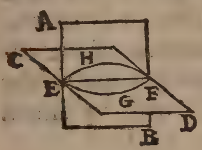
'This propoſition ſufficiently determines a 'plane, by the concouſe of two right lines, or by 'a triangle. I have alſo made uſe of it in *Op*-'*ticks*, to prove that objective parallel lines, 'which meet upon a Table, ought to be repre-'ſented by lines that concur in a point.

PROP.

PROPOSITION III.

A THEOREM.

The common section of two planes is one right line.



IF the planes AB and CD cut each other, their common section EF will be one right line. For if not, take two points common to both planes, as E and F; and draw a right line from the point E to the point *F* upon the plane AB, which suppose to be EHF. Draw likewise upon the plane CD a right line from the same point E to F; and if it be not the same with the former, suppose it to be EG*F*.

Demonstration.

These lines drawn upon two planes are two different lines, and enclose space; which is contrary to the 12. *Axiom. of the* 1. Therefore they will make but one right line, which being in both the planes will be their common section.

The *USE.*

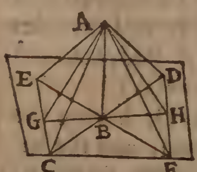
'This is a fundamental proposition, suppos'd in 'divers parts of the *Mathematicks*, though it 'be not always quoted. Particularly, it is ta-'ken for granted in *Gnomonicks*, when the hour-lines

'lines are repreſented upon *Dials*, by marking 'only the common ſection of their plane, and 'that of the wall.

PROPOSITION IV.

A Theorem.

If a line be perpendicular to two others that cut each other, it will alſo be perpendicular to the plane of the ſame lines.



IF the line AB be perpendicular to the lines CD and EF, which cut each other at the point B, ſo that the angles ABC, ABD, ABE, and ABF, be right angles, (which cannot conveniently be repreſented upon a plate,) it will be alſo perpendicular to the plane of the lines CD and EF, *i.e.* to all the lines that ſhall be drawn upon the ſame plane through the point B; as, for example, the line GBH. Let the lines BC, BD, BE, and BF, be equal, and draw the lines EC, DF, AC, AD, AE, AF, AG, and AH.

Demonſtration.

The four triangles ABC, ABD, ABE, and ABF, have each a right angle at the point B; and the ſides BC, BD, BE, and BF equal, with the

the ſide AB common to all. Therefore their baſes AC, AD, AE, and AF, are equal, (*by the* 4. 1.)

2. The triangles EBC and DBF will be in all reſpects equal, having their ſides BC, BD, BE, and BF, equal; and the angles CBE and DBF, being oppos'd at the top, equal: therefore the angles BCE, BDF, BEC, and BFD, will be equal, (*by the* 4. 1.) and alſo the baſes EC and DF.

3. The triangles GBC, and DBH, having the oppoſite angles CBG, and DBH, equal; as alſo the angles BDH, and BCG, and the ſides BC and BD; the ſides BG and BH, CG and DH, will be alſo equal, (*by the* 26. 1.)

4. The triangles ACE and AFD, having the ſides AC, AD, AE, and AF, equal, and the baſes EC and DF alſo equal; the angles ADF and ACE will be equal, (*by the* 8. 1.)

5. The triangles ACG and ADH have the ſides AC and AD, CG and DH equal, with the angles ADH and ACG; therefore their baſes AG and AH are equal.

Laſtly, the triangles ABH and ABG have all their ſides equal; therefore (*by the* 8. 1.) the angles ABG and ABH will be equal, and the line AB perpendicular to GH Accordingly the line AB will be perpendicular to any line drawn through the point B upon the plane of the lines CD and EF, which I call being perpendicular to their plane.

The

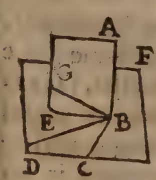
The USE.

'This Proposition occurs very oft in the first 'book of *Theodosius*: for example, to demon-'strate that the *Axis* of the world is perpendi-'cular to the plane of the *Equinoctial.* In like 'manner in *Gnomonicks*, tis demonstrated by 'this proposition, that the *Equinoctial line* in '*Horizontal Dials* is perpendicular to the *Me-'ridian.* Nor is it less useful in other *Mathe-'matical* Treatises; as those concerning *Astro-'labes*, and the cutting of pretious stones.

PROPOSITION V.

A THEOREM.

If a line be perpendicular to three others, which cut each other at the same point, they will be all three in the same plane.



If the line AB be perpendicular to three lines BC, BD, and BE, which cut each other at the same point B, the lines BC, BD, and BE, are in the same plane. Suppose the plane AE to be that of the lines AB and BE, and CF that of the lines BC and BD. If BE be the common section of both the planes, it will be in the plane

 of

of the lines BC and BD, as was asserted : but if BE be not, let BG be their common section.

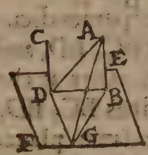
Demonstration.

AB is perpendicular to the lines BC and BD, therefore it is perpendicular to their plane CF, (*by the* 4.) and (*by defin.* 3.) *AB* will be perpendicular to BG. But it is also suppos'd perpendicular to BE ; therefore the angles ABE and ABG are right angles, and consequently equal, though one be part of the other. Therefore the two planes can have no other common section but BE, BE is therefore in the plane CF.

PROPOSITION VI.

A THEOREM.

Lines that are perpendicular to the same plane, are parallel.



IF the lines AB and CD be perpendicular to the same plane EF, they will be parallel. 'Tis evident, that the internal angles ABD and BDC are right angles; but that is not enough; it remains to be prov'd, that *AB* and CD are in the same plane. Draw DG perpendicular to *BD*, and equal to *AB*; draw also the lines *BG*, *AG*, and AD. *De-*

Demonstration.

The triangles ABD and BDG have the sides AB and DG equal, and BD common to both: and the angles ABD and BDG are right angles, therefore their bases AD and BG are equal, (*by the* 4. 1.) Further, the triangles ABG and ADG have all their sides equal: therefore the angles ABG and ADG are equal; and ABG being a right angle, because AB is perpendicular to the plane, ADG is also a right angle. Therefore the line DG is perpendicular to three lines CD, DA, and DB, which consequently are in the same plane, (*by the* 5.) but the line AB is in the plane of the lines AD and DB, (*by the* 2.) therefore AB and CD are in the same plane.

Coroll. Two parallel lines are in the same plane.

The USE.

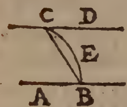
'By this proposition we demonstrate, that the 'hour-lines, in all planes that are parallel to 'the *Axis* of the world, as the Polars, Meridional, and others, are parallel among themselves.

PROP.

PROPOSITION VII.

A THEOREM.

A line drawn from one parallel to another, is in the same plane with them.



THE line CB, being drawn from the point B of the line AB to the point C of its parallel CD, is (I ſay) in the plane of the lines AB and CD.

Demonſtration.

The parallels AB and CD are in the ſame plane: in which if you draw a right line from the point B to the point C, it will be the ſame with CB; otherwiſe two right lines would encloſe ſpace, contrary to the 12. *Axiom. of the 1.*

PROPOSITION VIII.

A THEOREM

If one of two parallel lines be perpendicular to a plane, the other will be ſo alſo.

IF of the two parallel lines AB and CD, [*ſee fig. prop. 6.*] the one AB be perpendicular to the plane

plane EF, the other CD will be ſo alſo. Draw the line DB: ſince the angle ABD is a right angle, and AB and CD are ſuppos'd to be parallels, the angle CDB will be a right angle, (*by the* 29. 1.) therefore if I can prove, that the angle CDG is alſo a right angle, it will follow (*by the* 4.) that CD is perpendicular to the plane EF. Make a right angle BDG, and take DG equal to AB; then draw the lines BG and AG.

Demonſtration.

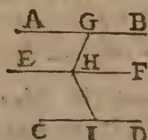
The triangles ABD and BDG have the ſides AB and DG equal, with the ſide BD common to both; and the angles ABD and BDG are right angles: therefore (*by the* 4. 1.) their baſes AD and BG are equal. The triangles ADG and ABG have all their ſides equal, therefore (*by the* 8. 1.) the angles ADG and *ABG* are equal: But the latter is a right angle, becauſe *AB* is ſuppos'd to be perpendicular to the plane EF, therefore the angle ADG is a right angle; and the line DG being perpendicular to the lines *DB* and DA, will be perpendicular to the plane of the lines *DB* and DA, which is the ſame in which are the parallels *AB* and CD. Therefore the angle GDC is a right angle, (*by defin.* 3.)

PROP.

PROPOSITION IX.

A Theorem.

Lines, that are parallel to a third, are also parallel among themselves, though not all in the same plane.



If the lines *AB* and *CD* are parallel to the line EF, they will be parallel to each other, though all the three lines be not in the same plane. Upon the plane of the lines *AB* and EF draw the line HG perpendicular to *AB*; which will be also perpendicular to EF, [*by the Lemma after the* 26. 1.] In like manner upon the plane of the lines EF and CD draw the line HI perpendicular to EF and CD.

Demonstration.

The line EH being perpendicular to the lines GH and HI, is so also to the planes of the lines HG and HI, (*by the* 4.) therefore (*by the* 8.) the lines AG and CI are perpendicular to the plane of the lines HG and HI, and [*by the* 6.] parallel to each other.

The USE.

'This proposition is frequently used in *Perspectives*, to determine the representation of 'parallel

'parallel lines upon a table; as also in the cut-
'ting of precious Stones, to prove the sides of
'the Pannels to be parallel among themselves,
'because they are so to a line in a different
'plane. In *Gnomonicks* likewise we are some-
'times oblig'd to prove, that the Vertical cir-
'cles ought to be describ'd on walls by perpen-
'dicular lines; because the lines, that are the
'common sections of them and the walls, are
'parallel to a line drawn from the *Zenith* to the
'*Nadir*.

PROPOSITION X.

A THEOREM.

If two lines, which concur, are parallel to two others concurring, of a different plane, they will make equal angles.

IF the lines AB and CD, AE and CF be parallel, though they be not all four upon the same plane, yet the angles BAE and DCF will be equal. Let the lines AB and CD, AE and CF be equal, and draw the lines BE, DF, AC, BD, and EF.

Demonstration.

The lines AB and CD are suppos'd to be both parallel and equal, therefore [*by the* 33.1.] the

the lines AC and BD are parallel and equal, as alſo AC and EF; and [*by the preceding*] BD and EF are parallel, and equal, and conſequently [*by the* 33. 1.] BE and DF will be alſo parallel and equal. Therefore the triangles BAE and DCF have all their ſides equal: and [*by the* 8. 1.] the angles BAE and DCF will be equal.

Coroll. Many the like propoſitions might be made, which would not be altogether unuſeful; as for example, if upon a parallel plane the line CD be drawn parallel to the line AB, and the angles BAE and DCF be equal, the lines AE and CF will be parallel.

The USE.

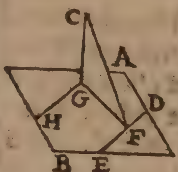
'By this propoſition we demonſtrate, that 'the angles made by the planes of the hour-'circles with a plane parallel to the *Equator*, 'are equal to the angles made by them with the 'plane of the *Equator*.

PROP.

PROPOSITION XI.

A PROBLEM.

To draw a perpendicular to a plane from a point given out of the plane.



If you desire to draw a perpendicular from the point C to the plane AB, draw the line EF at pleasure, and CF perpendicular to it, [*by the* 12. 1.] And again [*by the* 11.1.] upon the plane AB draw FG perpendicular to ED, and CG perpendicular to FG. I say, CG will be perpendicular to the plane AB. Draw GH parallel to FE.

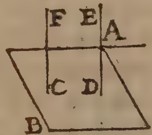
Demonstration.

The line EF being perpendicular to the lines CF and FG, will be perpendicular to the plane CFG, [*by the* 4.] and HG being parallel to EF, will be also perpendicular to the same plane, (*by the* 8.] And because CG is perpendicular to the lines GF and GH, it will be perpendicular to the plane AB, [*by the* 4.]

PROPOSITION XII.

A PROBLEM.

To draw a perpendicular to a plane through a point of the same plane.



TO draw a perpendicular to the plane AB through the point C, draw from the point E, taken at pleasure out of the plane, the line ED perpendicular to the same plane, [*by the* 11.] Draw also [*by the* 31. 1.] CF parallel to DE. CF will be perpendicular to the plane AB, [*by the* 8.]

PROPOSITION XIII.

A THEOREM.

Two lines perpendicular to a plane cannot be drawn through the same point.

IF the two lines CE and CD, drawn through the same point C, were perpendicular to the plane AB, and CF the common section of the planes of those lines, with the plane AB; the angles ECF and DCF

DCF would be both right angles, which is impossible.

I adde, that two perpendiculars DC and DF to the plane AF cannot be drawn from the same point D: for having drawn the line CF, there would be two right angles, DCF and DFC, in the same triangle, *contrary to the 32. 1.*

The USE.

'This proposition is necessary to show, that a 'perpendicular to a plane was sufficiently de-'scrib'd, in as much as but one such can be 'drawn through the same point.

PROPOSITION XIV.

A THEOREM.

Planes, to which the same line is perpendicular, are parallel.

IF the line AB be perpendicular to the planes AC and BD, they will be parallel, *i. e.* they will in all places be equally distant from each other. Draw the line DC parallel to AB, [*by the* 31. 1.] and joyn the lines BD and AC.

Demonstration.

AB is suppos'd to be perpendicular to the

 planes

planes AC and BD, therefore the line CD, which is parallel to it, will be alſo perpendicular to them, [*by the* 8.] and conſequently the angles B and D, A and C, will be right angles; and [*by the* 28. 1.] the lines AC and BD will be parallels, and the figure ABDC a parallelogram. Therefore the lines AB and CD are equal, [*by the* 34.1.] *i. e.* the planes in the points A and C, B and D, are equally diſtant. Accordingly the line CD may be drawn through any other point whatſoever; therefore the planes AC and BD are equally diſtant in all places the one from the other.

The USE.

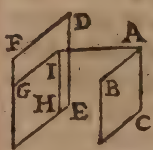
'*Theodoſius* demonſtrates the circles, that have 'the ſame poles, as the *Equator*, and the two '*Tropicks*, to be parallel, becauſe the *Axis* of 'the world is perpendicular to their planes.

PROP.

PROPOSITION XV.

A Theorem.

If two lines, meeting at a point, be parallel to two lines of another plane, , the planes of those lines will be parallel.



IF the lines AB and AC be parallel to the lines DE and DF, which are in another plane, the planes BC and FE are parallel. Draw AI perpendicular to the plane BC, [*by the* 11.] and GI and IH parallel to *F*D and DE: they will be also parallel to the lines AB and AC, [*by the* 9.]

Demonstration.

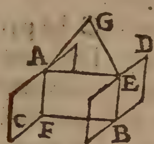
The lines AB and GI are parallel, and the angle IAB is a right angle, AI being perpendicular to the plane BC: therefore [*by the* 29.1.] the angle AIG is a right angle, as also the angle AIH. Therefore [*by the* 4.] the line AI is perpendicular to the plane GH; and being also perpendicular to the plane BC, the planes BC and GH, or *FE*, will be parallel, [*by the* 14.]

PROP.

PROPOSITION XVI.

A THEOREM.

If a plane cut two others which are parallel, their common sections will together with them be parallel.



IF the plane AB cut two other parallel planes, AC and BD; I say, their common sections AF and BE will be parallel. For if not, being continu'd they would at length concur, *e. g.* at the point G.

Demonstration.

The lines AF and BE are upon the planes AC and BD; and therefore [*by the* 1.] can never be either above, or below it; therefore if they concur at the point G, the planes must do so likewise, and consequently they would not be parallel, which is contrary to what was suppos'd.

The USE.

'By this Proposition we demonstrate, in the 'Treatise of *Conick* and *Cylindrick* Sections, 'that if the *Cone* or *Cylinder* be cut by a plane 'parallel to its base, the sections are circular. 'By the same we describe *Astrolabes*; and 'prove in *Gnomonicks*, that the angles, which the

'the hour-circles make with a plane parallel to 'a great circle, are equal to those which they 'make in the circle it self; and again in *Per-'spectives*, that the Images of the objective lines 'perpendicular to the table, concur at the point 'of sight.

PROPOSITION XVII.

A THEOREM.

Two lines are divided proportionally by parallel planes.

IF the lines AB and CD be be divided by parallel planes, I say, AE will have the same proportion to EB as CF to FD. Draw the line AD, passing through the plane EF at the point G; Draw also AC, BD, FG, and GE.

Demonstration.

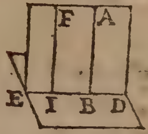
The plane of the triangle ABD cuts the three planes, therefore (*by the* 16.) the sections BD and EG are parallel; and (*by the* 2. 6.) AE has the same proportion to EB, as AG to *GD*. In like manner the plane of the triangle ADC cuts the planes EF and AC, therefore the sections AC and *FG* are parallel; and *FC* has the same proportion to *FD* as *AG* to *GD*, *i. e.* as AE to EB.

PROP.

PROPOSITION XVIII.

A THEOREM.

If a line be perpendicular to a plane, all the planes, in which that line is found, are perpendicular to the same plane.



IF the line AB be perpendicular to the plane ED, all the planes in which it is found will be perpendicular to the plane ED. Suppose AB to be in the plane AE, having for a common section with the plane ED the line BE; to which draw a perpendicular FI.

Demonstration.

The angles ABI and BIF are right angles, therefore the lines AB and *FI* are parallel; and (*by the* 8.) *FI* will be perpendicular to the plane ED. Therefore the plane AE will be perpendicular to the plane ED, (*by def.* 4.)
"The same may be prov'd of the plane AD.

The USE.

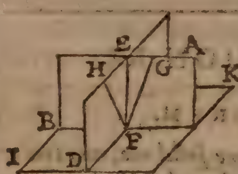
'The first Proposition in *Gnomonicks*, which 'may pass for a Fundamental one, is built upon 'this proposition; which is also frequently 'made use of in *Spherical Trigonometry*, in *Per-spectives*,

'*spectives*, and generally in all those Treatises 'which are oblig'd to consider divers planes.

PROPOSITION XIX.

A THEOREM.

If two planes cutting each other be perpendicular to another, their common section will be perpendicular to the same.



IF the planes AB and ED, which cut each other, be perpendicular to the plane IK, their common section EF is also perpendicular to the plane IK.

Demonstration.

If EF be not perpendicular to the plane IK, upon the plane AB draw the line GF perpendicular to the common section BF: and the plane AB being perpendicular to the plane IK, the line GF will be perpendicular to the same plane. Draw likewise FH perpendicular to the common section DF; it will be also perpendicular to the plane IK. We shall have therefore two perpendiculars to the same plane, drawn through the same point F, (*contrary to the*

13. *Propos.*) it must therefore be granted that EF is perpendicular to the plane IK.

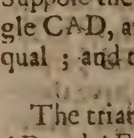
The USE.

'By this proposition we demonstrate, that the 'circle which passes through the Poles of the 'world and the *Zenith* is the *Meridian*, and 'cuts all the diurnal arches into two equal 'parts; and that the Stars spend as much time 'in their motions from their risings to this cir-'cle, as from the circle to their settings.

PROPOSITION XX.

A THEOREM.

If three plain angles make one solid one, any two of them ought to be greater than the third.



IF the angles BAC, BAD, and CAD, make the solid angle A, and the angle BAC be the greatest angle; the two others, taken together, are greater than BAC. Suppose the angle CAE to be equal to the angle CAD, and the lines AD and AE to be equal; and draw the lines CEB, CD, and BD.

Demonstration.

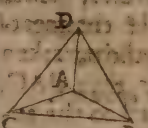
The triangles CAE and CAD have the sides AD and AE equal, and the side AC common to both,

both; and the angles CAD and CAE equal: therefore (*by the* 4. 1.) their bases CD and CE are equal. But the sides CD and DB are greater than the side CB alone, (*by the* 20. 1.) therefore taking away the equal lines CD and CE, the line BD will be greater than BE. Further, the triangles BAE and BAD have the sides AE and AD equal, and the side BA common, and the base BD greater than the base BE: therefore (*by the* 18. 1.) the angle BAD is greater than the angle BAE; adding therefore the equal angles CAD and CAE, the angles BAD and CAD will be greater than the angles CAE and BAE, *i. e.* the angle BAC.

PROPOSITION XXI.

A THEOREM.

All the plain angles, that make one solid angle, are less than four right angles.



IF the plain angles BAC, BAD, and CAD, make the solid angle A, they will be less than four right angles. Draw the lines BC, BD, and CD, and you will have a pyramid, whose base is the triangle BCD.

Demonstration.

The solid angle at the point B, has the angles ABC and ABD greater than that of the

base alone *CBD*. In like manner ACB and ACD are greater than BCD alone, and the angles ADC and ADB are greater than CDB alone. But all the angles of the base are equal to two right angles, therefore the angles ABC, ABD, ACB, ACD, ADC, and ADB, are greater than two right angles. And because all the angles of the three triangles *BAC*, *BAD*, and CAD, are equal to six right angles; taking away more than two right angles, there will remain less than four, for the angles made at the point A. But if the solid angle A consist of more than three plain angles, so that the base of the pyramid be a polygon, it may be divided into triangles; and the computation being made, you will find, that all the plain angles, which make up the solid one, are always less than four right angles.

The USE.

'These two propositions show when many 'plain angles may make up one solid one, which 'is often necessary in the treatises of cutting of 'Stones, and in the following propositions.

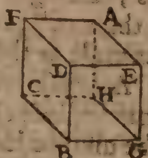
The 22, and 23. Propositions are of no use.

PROP.

PROPOSITION XXIV.

A Theorem.

If a solid body be terminated by parallel planes, the opposite sides will be similar and equal parallelograms.



If the solid AB be terminated by parallel planes, the opposite superficies's will be similar and equal parallelograms.

Demonstration.

The parallel planes AC and *BE* are cut by the plane *FE*; therefore their common sections are parallel, (*by the* 16.) and so likewise DF and AE; therefore AD will be a parallelogram. After the same manner I may demonstrate, that AG, FB, CG, and the rest, are parallelograms. I adde, that the opposite parallelograms, *e. g.* AG and *FB*, are similar and equal. The lines AE and EG are parallel to the lines FD and DB: therefore the angles AEG and *FDB* are equal, (*by the* 10.) Accordingly I may demonstrate all the sides and all the angles of the opposite parallelograms to be equal, therefore the parallelograms are similar and equal.

PROP.

PROPOSITION XXV.

A THEOREM.

If a Parallelepipedon be divided by a plane parallel to one of its planes, the two solid bodies which arise by that division, will have the same proportion as their bases.

IF the parallelepipedon AB be divided by the plane CD, which is parallel to the planes AF and BE, the solid AC will have the same proportion to BD as the base AI to the base DG. Suppose the line AH, which shows the hight of the figure to be divided into as many equal parts as you please; for example, ten thousand; which we may take as indivisibles, *i.e.* without reflecting upon the possibility of their being further subdivided. Suppose also so many superficies's parallel to the base AI, as there are parts in the line AH; I have describ'd only one OS: so that the solid AB be compounded of all those superficies's of the same thickness, as a Ream of paper is compounded of all its sheets and quires laid one upon another. 'Tis evident that so the solid AC will be compounded of ten thousand super-

superficies's equal to the base AI, (*by the preceding*,) and the solid *DB* will contain ten thousand superficies's equal to the base DG.

Demonstration.

Every superficies of the solid AC has the same proportion to any of the superficies's of the solid DB, as the base AI to the base DG; because they are every one of them equal to their bases: therefore (*by the* 12. 5.) all the superficies's of the solid AC, taken together, will have the same proportion to all those of the solid DB, as the base AI to the base DG. But all the superficies's of the solid AC make up the solid AC, which has no other parts but those superficies's; and all the superficies's of the solid *DB* are nothing else but the solid DB; therefore the solid AC has the same proportion to the solid DB, as the base AI to the base DG.

The USE.

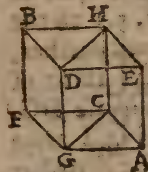
This is *Cavalerius*'s demonstration; which is very clear, provided it be used as it ought; and that the line, by which is measur'd the thickness of the superficies's, be taken in the same respect in both the terms. I shall make use of it hereafter, to render some intricate and perplex'd demonstrations more facil and clear.

PROP.

PROPOSITION XXVI.

A THEOREM.

A parallelepipedon is divided into two equal parts by the diagonal plane.



Suppose the parallelepipedon AB to be divided by the plane CD, drawn from one angle to another: I say it will be divided into two equal parts. Divide the line AE into as many parts as you please, and draw so many planes parallel to the base AF; each of those planes is a parallelogram equal to the base *AF*, (*by the* 24.)

Demonstrasion.

All the parallelograms, that can be drawn parallel to the base *AF*, are divided into two equal parts by the plane CD: for the triangles which are form'd on both sides the plane CD, have their base common, in each equal to CD, and their sides equal, being those of a parallelogram. But tis evident, that the parallelepipedon is nothing else but those parallelograms, which are each divided into two equal triangles: therefore the parallelepipedon is divided into two equal parts by the plane CD.

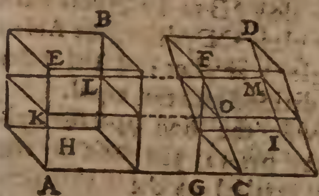
'The

The 27. and 28. Propositions are of no use according to this way of demonstrating.

PROPOS. XXIX, XXX, XXXI.

A THEOREM.

Parallelepipedons of the same hight, having the same or equal bases, are equal.



IF the Parallelepipedons AB and CD be of the same hight according to the perpendiculars AE and FG, and have the same or equal bases AH and CI, they will be equal. Suppose the two bases to be set upon the same plane; since their perpendiculars are equal, the bases EB and FD will be in the same plane, which will be parallel to the plane of the bases AH and CI. Suppose then the line AE or FG to be divided into as many equal parts as you please, *e. g.* ten thousand, and according to them so many superficies's or planes drawn of the same thickness: I have describ'd only one for all, as K or M. Each superficies will form in these solids a parallel plane, similar and equal to the base, *(by the* 24.) as KL, OM;

 and

and there will be as many in one solid as in the other ; because their thickness, which I take perpendicularly according to their *respective* hights, is equal.

Demonstration.

The base AH has the same proportion to the base CI , as each plane KL to OM. But they being equal in number in both, all the antecedents (*by the* 15.5.) will have the same proportion to all the consequents, *i. e.* the whole solid AB to the whole solid CD, as the base AH to the base CI. But tis suppos'd that the bases are equal, therefore the solids are equal.

Coroll. To find the solidity of a parallelepipedon, tis usual to multiply the base by the hight taken perpendicularly, because that perpendicular shows how many superficies's equal to the base are contain'd in it. As for example, if I take a foot for my indivisible measure, *i. e.* which I will not afterwards subdivide ; if the base contain twelve feet square, and the perpendicular hight ten, I shall have an hundred and twenty cubick feet for the solidity of the body AB. For the hight containing ten feet, I may make ten parallelograms equal to the base, having each a foot in thickness ; but the base with one foot in thickness makes twelve cubick feet : the whole therefore will make an hundred and twenty, if the hight contain ten feet.

PROP.

PROPOSITION XXXII.

A THEOREM.

Parallelepipedons of the same hight are in the same proportion as their bases.

I Have prov'd this proposition in the preceding, demonstrating, that the parallelepipedon AB has the same proportion to the parallelepipedon CD, as the base AH to the base CI. *(See fig. preced.)*

Coroll. Parallelepipedons that have equal bases, are in the same proportion as their hights. As the parallelepipedons AB and AL, whose perpendicular hights are AK and AE. For if you divide the hight AK into as many aliquot parts as you please, and AE into as many as it contains equal to the former, and draw, according to each part, planes parallel to the base; as many as AE contains of the aliquot parts of AK, so many will the solid AB contain of the superficies's equal to the base, which are the aliquot parts of the solid AL; therefore (*by defin.* 5. 5.) the solid AB will have the same proportion to the solid AL, as the hight AE to the hight AK.

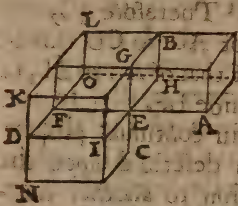
The USE.

'The three preceding propositions contain 'almost all the ways of measuring parallelepi-'pedons, and may be esteem'd as first principles 'for that purpose. 'Tis after the same manner 'also that we take the dimensions of the solidity 'of Walls, by multiplying their bases by their 'hights.

PROPOSITION XXXIII.

A THEOREM.

Similar parallelepipedons are in the triplicate proportion of their homologous sides.



IF the parallelepipedons AB and CD be similar; *i. e.* if all the planes of one be like those of the other; and all their angles equal, so that they may be plac'd in a right line, *i. e.* that AE and EF, HE and EI, GE and EC, may make right lines; and AE has the same proportion to EF as HE to EI, and as GE to EC: I say, that here are four solids in continual proportion according to the proportion of the side EA to that, which is homologous to it, EF or DI.

De-

Demonstration.

The parallelepipedon AB has the ſame proportion to EL, of the ſame hight, as the baſe AH to the baſe EO, (*by the* 32.) But the baſe AH has the ſame proportion to the baſe EO, as AE to EF, (*by the* 1.6.) In like manner, the proportion of the ſolid EL to the ſolid EK, is the ſame with that of the baſe EO to the baſe ED, *i.e.* that of HE to EI. And laſtly, the ſolid EK has the ſame proportion to the ſolid EN, as the hight GE to the hight EC, (*by the coroll. of the* 32.) or (taking the line EF for their common hight) as the baſe GI to the baſe CI, *i.e.* as GE to EC. But the proportion of AE to EF, of HE to EI, and of GE to EC, was ſuppos'd to be the ſame; and conſequently, the ſolid AB has the ſame proportion to EL as EL to EK, and as EK to CD. Therefore (*by defin.* 11. 5.) the proportion of AB to CD will be the triplicate proportion of that of AB to EL, or of AE to its homologous ſide EF.

Coroll. If four lines be in continual proportion, the parallelepipedon deſcrib'd upon the firſt, has the ſame proportion to another ſimilar parallelepipedon deſcrib'd upon the ſecond, as the firſt to the fourth; for the proportion of the firſt to the fourth, is the triplicate proportion of that of the firſt to the ſecond.

The U S E.

'You may perceive by this propoſition that 'that

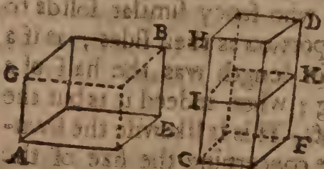
'that famous problem of the duplication of the 'Cube, propos'd by the Oracle, consists in find- 'ing two middle terms in continual proportion. 'For if you make the side of the first cube the 'first term, and the double of that the fourth; 'having found two middle proportionals, the 'cube describ'd upon the first line will have the 'same proportion to that describ'd upon the se- 'cond, as the first line to the fourth, *i. e.* as one 'to two. By this proposition also may be cor- 'rected their error, who fancy similar solids to 'have the same proportion as their sides; as if a 'cube of one foot in length was the half of a 'cube two foot long; when indeed it is but the 'eighth part thereof. This is likewise the foun- 'dation of the Rule concerning the size of the 'bores of Canons; and is applicable not only to 'bullets, but to all sorts of similar bodies. For 'example; should a man, about to build a Na- 'vy, and resolving to retain the same proportion 'in all his vessels, reason thus with himself; If 'a Ship of an hundred tun require fifty foot in 'Keel, another of two hundred tuns ought to 'have an hundred foot in Keel; he would be 'guilty of a great mistake: for instead of mak- 'ing a vessel twice as large as the former, he 'would make one eight times so much. He ought 'to assign to the second vessel somewhat less 'than sixty three feet.

PROP.

PROPOSITION *XXXIV.*

A THEOREM.

Equal parallelepipedons have their bases's and hights reciprocal, and those that have their bases and hights reciprocal are equal.



IF the parallelepipedons AB and CD be equal, their bases and hights will be reciprocal, *i. e.* the base AE will have the same proportion to the base CF, as the hight CH to the hight AG. Having made CI equal to AG, draw the plane IK parallel to the base CF.

Demonstration.

The parallelepipedon AB has the same proportion to CK, being of the same hight, as the base AE to the base CF, (*by the* 32.) But as AB to CK, so is CD to the same CK, because AB and CD are equal; and as CD to CK, which have both the same base, so is the hight CH to the hight CI, (*by the coroll, of the* 32. therefore as the base AE to the base CF, so is the hight CH to the hight CI or AG.

I add, that if the baſe AE has the ſame proportion to the baſe CF as the hight CH to the hight AG, the ſolids AB and CD will be equal.

Demonſtration.

AB has the ſame proportion to CK, being of the ſame hight, as the baſe AE to the baſe CF, (*by the* 32.) Alſo the hight CH has the ſame proportion to the hight CI or AG, as CD to CK: but we ſuppoſe that AE has the ſame proportion to CF, as CH to CI or AG; therefore the ſolid AB has the ſame proportion to the ſolid CK as the ſolid CD to the ſame CK, and conſequently the ſolids AB and CD are equal, (*by the* 9.5.)

The USE.

'This Reciprocation of the baſes and hights 'makes the ſolid very eaſie to be meaſur'd. And 'the propoſition ſeems to bear ſome analogy to 'the 14. *Prop. of the* 6. which aſſerts, That e-'quiangular and equal parallelograms have their 'ſides reciprocal; and the practice of the Rule 'of Three may be demonſtrated from both.

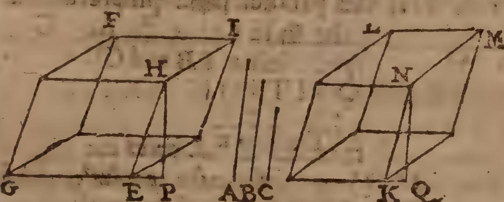
The 35. *Prop. may be omitted.*

PROP.

PROPOSITION XXXVI.

A THEOREM.

If three lines be in continual proportion, a parallelepipedon made of those three lines is equal to an equiangular parallelepipedon, which has all its sides equal to the middle line.



IF the lines A,B,C, be in continual proportion, the parallelepipedon EF made of those three lines, the side FI being equal to the line A, HE equal to B, and HI equal to C, is equal to the equiangular parallelepipedon KL, whose sides LM,MN, and KN, are each of them equal to the line B. From the points H and N draw the lines HP and NQ perpendicular to the planes of the bases; which lines will be equal, because the solid angles E and K are suppos'd equal, (so that if they could penetrate, neither would exceed the other,) and the lines EH

and KN are alſo ſuppos'd equal. Therefore the hights HP and NQ are equal.

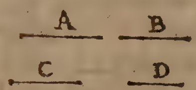
Demonſtration.

There is the ſame proportion of A to B, or of *FI* to LM, as of B to C, or LM to *HI*: therefore the parallelogram FH contain'd under *FI* and *IH* is equal to the parallelogram LN contain'd under LM and MN both equal to B, (*by the* 16.6.) therefore the baſes are equal. But the hights HP and NQ are alſo equal; therefore (*by the* 31.) the parallelepipedons are equal.

PROPOSITION XXXVII.

A THEOREM.

If four lines be proportional, the parallelepipedons deſcrib'd upon thoſe lines are proportional: and if the ſimilar parallelepipedons be proportional, their homologous ſides will be alſo proportional.



IF the line A has the ſame proportion to B as C to D, the ſimilar parallelepipedons, whoſe homologous ſides are the lines A, B, C, D, will be in the ſame proportion.

Demonſtration.

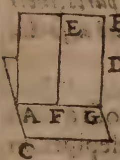
The parallelepipedon A is in the triplicate proportion to the parallelepipedon B, of that of

of the line A to the line B, or that of the line C to the line D. But the parallelepipedon C to the parallelepipedon D is also in the triplicate proportion of that of the line C to the line D, *(by the 33.)* Therefore the parallelepipedon A has the same proportion to the parallelepipedon B, as the parallelepipedon C to the parallelepipedon D.

PROPOSITION XXXVIII.

A THEOREM.

If two planes be perpendicular to each other, a perpendicular drawn from a point in one of the planes to the other will fall upon the common section.



IF, the planes AB and CD being perpendicular to each other, you draw from the point E in the plane AB a line perpendicular to the plane CD, it will fall upon the common section of the planes. Draw EF perpendicular to the common section AG.

Demonstration.

The line EF, perpendicular to AG, the common section of the planes, which are suppos'd to be perpendicular, will be perpendicular to the plane CD, *(by defin. 3.)* and because two lines

lines cannot be drawn from the point E perpendicular to the plane CD, (*by the* 13.) every perpendicular will fall upon the common section AG.

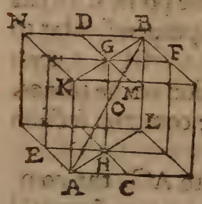
The USE.

'This proposition ought to have follow'd next 'after the 17*th*. because it respects solids in 'general. Tis of use to us in the Treatise of '*Astrolabes*, to prove that in the *Analemma* all 'the circles, perpendicular to the *Meridian*, 'ought to be markt by right lines.

PROPOSITION XXXIX.

A THEOREM.

If in a parallelepipedon be drawn two planes, which divide the opposite sides into two equal parts, their common section and the diameter will also divide each other into two equal parts.



Suppose the opposite sides of the parallelepipedon AB to be divided into two equal parts by the planes CD and EF, their common section GH and the diameter BA will equally divide each other at the point O. Draw the lines BG, GK, AH and HL. I shall prove first, that the

two

two first of these, BG and GK, (and so likewise AH and HL,) make but one right line. For the triangles DGB and KMG have their sides DB and KM equal, because they are the halves of equal sides; as also GD and GM. Further, DB and KM being parallel, the alternate angles BDG and GMK will be equal, (*by the* 29.1.) and therefore (*by the* 4. 1.) the triangles DBG and KGM will be equal in all respects, and consequently the angles BGD and KGM; and [*by the coroll. of the* 15. 1.] BG and GK make but one right line, as also LH and HA: therefore ALBK is one plane, in which are found both the diameter AB, and the common section of the planes GH. The plane ALBK cutting the parallel planes AN and CD, their common sections GH and AK will be parallel: and [*by the* 2. 6.] BG will have the same proportion to GK, as BO to OA; and therefore [*by the* 18. 5.] as BK to GK, so BA to BO; and [*by the* 4.6.] so GH or AK to OG. But BK is double to BG, therefore BA is double to BO and AK, equal to GH, double to GO. Therefore the lines GH and AB divide each other equally at the point O.

Coroll. 1. All the diameters are divided at the point O.

Coroll. 2. Here we may adde some Corollaries, which depend upon divers propositions. As for example, that triangular prisms of the same

ſame hight are in the ſame proportion as their baſes: for the parallelepipedons, of which they are the halves, are [*by the* 32.] in the ſame proportion as their baſes: therefore the halves of their baſes, and the halves of the parallelepipedons, *i. e.* the priſms, will be in the ſame proportion.

Coroll. 3. Polygon priſms of the ſame hight are alſo in the ſame proportion as their baſes, becauſe they may be reſolv'd into triangular ones, each of which will have the ſame proportion as their baſes.

Coroll. 4. The reſt of the propoſitions concerning parallelepipedons are alſo applicable to priſms; as for example, that equal priſms have their hights and baſes reciprocal; and that ſimilar priſms are in the triplicate proportion of that of their homologous ſides.

The USE.

'This propoſition may help us to find out the 'center of Gravity in parallelepipedons; and to 'demonſtrate ſome other propoſitions in the 'thirteenth and fourteenth books of *Euclid*,

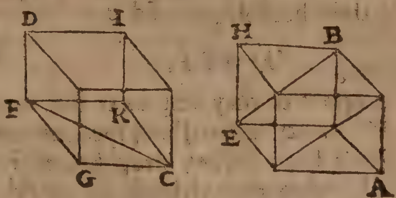
PROP.

PROPOSITION XL.

A Theorem.

A Prism, that has a parallelogram for its base double to the triangular base of another prism, and of the same hight, is equal to it.

LEt ABE and CDG be two triangular prisms, of the same hight; and the base of one the parallelogram AE, double to the triangle FGC, the base of the other prism: I say these prisms are equal. Suppose the parallelepipedons AH and GI were completed.



Demonstration.

'Tis suppos'd, that the base AE is double to the triangle FGC, but the parallelogram GK is double to the same triangle, [*by the* 34. 1.] therefore the parallelograms AE and GK are equal;

equal; and consequently, the parallelepipedons AH and GI, having the same bases and the same hights, are equal; and therefore the prisms that are the halves, [*by the* 26.] will be likewise equal.

THE

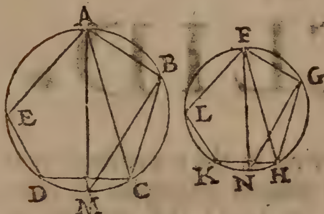
THE TWELFTH BOOK OF THE ELEMENTS OF EUCLID.

'E*uclid*, after having in the preceding books
'deliver'd the general principles of solid
'bodies, and explain'd the manner of
'measuring the most regular of them, that is,
'such as are terminated by plain superficies's;
'treats in this of such bodies as are contain'd in
'superficies's that are crooked, as the Cylinder,
'Cone, and Sphere: comparing one with the
'other, and giving rules, relating both to their
'solidity, and the manner of taking their dimen-
'sions. The book is of exceeding great use, be-
'cause in it we find the principles upon which
'the most learned *Mathematicians* have built so
'many famous demonstrations concerning the
'Cylinder, the Cone, and the Sphere.

PROPOSITION L.

A THEOREM.

Similar polygons, inscrib'd in circles, are in the same proportion as the squares of the diameters of the same circles.



IF the polygons ABCDE, and FGHKL, inscrib'd in circles, be similar, they will be in the same proportion as the squares of the diameters AM, FN. Draw the lines BM, GN, AC, and FH.

Demonstration.

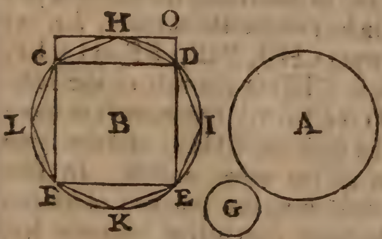
Tis suppos'd that the polygons are similar, that is to say, that the angles B and G are equal, and that AB has the same proportion to BC as FG to GH: from whence I infer, [*by the* 6.6.] that the triangles ABC and FGH are equiangular, and that the angles ACB and FHG are equal: so that likewise [*by the* 21.3.] the angles AMB and FNG are equal. But the angles ABM and FGN, being in a semicircle, are right angles, [*by the* 31.3.] and consequently, the triangles ABM and FGN are equiangular. There-

Therefore [*by the* 4. 6.] AB has the ſame proportion to FG, as AM to FN: and [*by the* 22. 6.] if two ſimilar polygons be deſcrib'd upon AB and FG, as thoſe that are propos'd; and two other ſimilar polygons upon AM and FN, which ſhall be two ſquares; the polygon ABCDE will have the ſame proportion to the polygon FGHKL, as the ſquare of AM to the ſquare of FN.

"This propoſition is neceſſary to demonſtrate "that which follows.

LEMMA.

If a certain quantity be leſs than a circle, a regular polygon may be inſcrib'd in the ſame circle greater than that quantity.



Suppoſe the figure A 'to be leſs than the circle B; a regular polygon may be 'inſcrib'd in 'the ſame circle, which ſhall be greater than the 'figure A. Let the figure G be the difference 'between the figure A and the circle, ſo that the figures

'figures A and G taken together, may be equal 'to the circle B. Inſcribe in the circle B the 'ſquare CDEF, [*by the* 6. 4.] and if the ſquare 'be greater than the figure A, we ſhall have 'what we wanted. If it be leſs, divide the four 'quarters of the circle CD, DE, EF, and FC, 'each into two equal parts at the points H, I, K, 'L, that ſo you may have an octogon. But if 'the octogon be ſtill leſs than the figure A, 'ſudivide its archs, and you will have a polygon 'of ſixteen ſides, afterwards of thirty two, and 'then of ſixty four. I ſay, at length you will have 'a polygon greater than the figure A, *i. e.* a po-'lygon whoſe difference from the circle is leſs 'than that of the figure A, that is, leſs than the 'figure G.

Demonſtration.

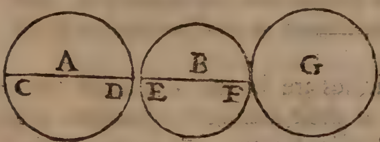
'The inſcrib'd ſquare is more than half of the 'circle, being half of the ſquare deſcrib'd about 'the circle; and in deſcribing the octogon you 'take more than half of the Remainder, *i. e.* of 'the four ſegments CHD, DIE, EKF, and CLF. 'For the triangle CHD is the half of the rect-'angle CO, [*by the* 34. 1.] therefore it is more 'than half of the ſegment CHD; and the ſame 'may be ſaid of all the other arches. In like 'manner, in deſcribing the polygon of ſixteen ſides, you take more than half of what was left 'of the circle; and ſo in all the others. There-'fore you will leave at laſt a leſs quantity than 'G. For tis evident, that two unequal quanti-
ties

'ties being propos'd, if you take away more 'than half of the greater, and afterwards more 'than half of what remains, and again more than 'half of what is ſtill left behind ; at length that 'which remains will be leſs than the ſecond 'quantity. Suppoſe the ſecond quantity to be 'contain'd in the firſt an hundred times: tis 'evident, that dividing the firſt into an hundred 'parts, in ſuch ſort, that the firſt part may have 'a greater proportion to the ſecond than two to 'one; the laſt will be leſs than the hundredth 'part: ſo that at laſt you will obtain a polygon, 'which will be leſs exceeded by the circle, than 'the circle exceeds the figure A; that is to ſay, 'that what will remain of the circle, when the 'polygon is taken away, will be leſs than G. 'Therefore the polygon will be greater than the 'figure A.

PROPOSITION II.

A THEOREM,

Circles are in the ſame proportion as the ſquares of their diameters.



I Prove, that the circles A and B are in the ſame proportion, as the

the ſquares of CD and EF. Suppoſe the figure G to have the ſame proportion to the circle B, as the ſquare of CD to the ſquare of EF: if the figure G be leſs than the circle A, [*by the preceding Lemma*,] a regular polygon may be inſcrib'd in the circle A greater than G. Let a ſimular regular polygon be alſo inſcrib'd in the circle B.

Demonſtration.

The polygon of the circle A will have the ſame proportion to the polygon of B, as the ſquare of CD to the ſquare of EF, *i. e.* the ſame as G to the circle B; but the quantity G is leſs than the polygon inſcrib'd in A: accordingly therefore [*by the* 14. 5.] the circle B muſt be leſs than the polygon inſcrib'd in it, which is manifeſtly falſe. It muſt therefore be granted that the figure G, being leſs than the circle A, cannot have the ſame proportion to the circle B, as the ſquare of CD to the ſquare of EF; and conſequently, that the circle A cannot have a greater proportion to the circle B, than the ſquare of CD to the ſquare of EF: Nor can it be ſaid to have a leſs; for then the circle B would have a greater proportion to the circle A, and the ſame demonſtration would be applicable to it.

Coroll. 1. Circles are in the duplicate proportion of that of their diameters; becauſe the ſquares being ſimilar figures, are in the duplicate

cate proportion of that of their ſides, [*by the* 20.6.]

Coroll. 2. Circles are in the ſame proportion as the ſimilar polygons, that are inſcrib'd in them.

Coroll. 3. This ought to be well obſerv'd as a general rule: When ſimilar figures, being inſcrib'd in others, ſo that they may approach ſtill nearer and nearer to them, and at laſt degenerate into the figures themſelves, are in the ſame proportion; the figures that contain them are alſo in the ſame proportion. What I would ſay is this; That ſimilar regular polygons, inſcrib'd in divers circles, are always in the ſame proportion as the ſquares of the diameters; and being made of more ſides, ſo as to approach ſtill nearer and nearer to the circles, they ſtill retain the ſame proportion; and the circles themſelves are in the ſame proportion as the ſquares of their diameters. This manner of meaſuring round bodies, by inſcribing in them others, is of great uſe.

The *USE.*

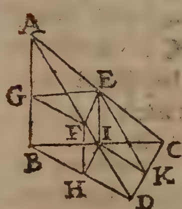
'This being a very general Propoſition, enables us to argue about circles in the ſame manner as we do of ſquares. For example, we ſay '[*in the* 47. 1.] That in a rectangle triangle the 'ſquare of the baſe alone is equal to the ſquares 'of both the ſides taken together. We may ſay 'the ſame of circles, *i. e.* That the circle, deſcrib'd

'ſcrib'd upon the baſe of a rectangle triangle, is 'equal to the circles, whoſe diameters are the 'ſides. And in the ſame manner may we aug-'ment or diminiſh circles, according to what 'proportion we pleaſe. We prove alſo by it in '*Opticks*, that the light decreaſes in the dupli-'cate proportion of that of the diſtances of the 'lucid bodies.

PROPOSITION III.

A THEOREM.

Every Pyramid, whoſe baſe is triangular, may be divided into two equal priſms, which make up more than half of the pyramid; and into two equal pyramids.



IN the pyramid ABCD may be found two equal priſms, EBFI, and EHKC, which will be greater than half the pyramid. Divide the ſix ſides of the pyramid equally at the points G,F,E,I, H,K, and draw the lines EG, GF, FE, EI, HI, FH, IK, and EK.

Demonſtration.

In the triangle ABD, AG has the ſame proportion to GB as AF to FD, becauſe AB and

AD

AD are equally divided in G, and F; therefore [*by the* 2.6.] GF and BD are parallels; and GF will be the half of BD, *i. e.* equal to BH. In like manner, GE and BI, FE and HI, will be parallels, and equal: and [*by the* 15.11.] the planes GFE and BHI will be parallel, and consequently EBFI will be a prism. The same may be said of the figure HEKF, which will be also a prism equal to the other, [*by the* 40.11.] the parallelogram base HIKD being double the triangular BHI, [*by the* 41.1.]

Secondly, I say, the pyramids AEFG, and ECKI, are similar and equal.

Demonstration.

The triangles AFG and FDH are equal, [*by the* 8.1.] as also FDH and EIK; and likewise AGE, and EIC, and so of all the other triangles of the pyramids: therefore the pyramids are equal, (*by defin.*10.11.) They are also similar to the great pyramid ABDC: for the triangles AGE and ECI are similar, (*by the* 2.6.) the lines GE and BC being parallels; and the like may be demonstrated of all the other triangles of the lesser pyramids.

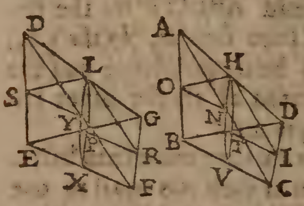
Lastly, I say the prisms are more than half of the first pyramid. For if each was equal to one of the lesser pyramids, both would be equal to the half of the greater pyramid. But they are each of them greater than one of those pyramids; as the prism GHE contains the pyramid

G3HI, and somewhat more; and that pyramid is equal and similar to the others, having all their triangles equal and similar to those of the pyramid AGFE, as may be easily prov'd by the parallelism of their sides: from whence I infer, that the two prisms taken together are greater than the two pyramids, and consequently greater than half of the great pyramid.

PROPOSITION IV.

A Theorem.

If two triangular pyramids of the same hight be divided into two prisms and two pyramids, and the latter pyramids subdivided after the same manner; all the prisms of one pyramid will have the same proportion to all those of the other, as the base of one pyramid to the base of the other.



If the two pyramids ABCD, DEFG, of the same hight, and having triangular bases, be divided into two prisms and two pyramids, according to the method laid down in the third proposition; and the two lesser pyramids be subdivided after the same manner, and so

so in order, that, having made as many divisions of one as of the other, you have the same number of prisms in both; I say, that all the prisms of one will have the same proportion to all the prisms of the other, as their bases.

Demonstration.

The pyramids being of the same hight, the prisms, produc'd by the first division, will have also the same hight, because they have each the half of that of their pyramids. But prisms of the same hight are in the same proportion as their bases, (*by the coroll. of the* 39. 11.) The bases BTV and EPX are similar to the bases BDC and EGF; and having for their sides the half of those great bases, they can make but the fourth part of them, but they are in the same proportion as the great bases are; therefore the first prisms will have the same proportion as the great bases. After the same manner I may prove that the prisms produc'd by the second division, *i.e.* of the lesser pyramids, will be in the same proportion as the bases of those lesser pyramids, which are in the same proportion as the great bases. Therefore all the prisms of one have the same proportion to all the prisms of the other, as the base to the base.

The USE.

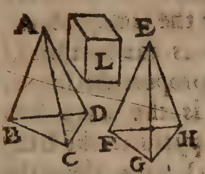
"These two propositions are necessary to "compare pyramids together, and to take their "dimensions.

PROP.

PROPOSITION V.

A THEOREM.

Triangular pyramids of the same hight are in the same proportion as their bases.



THE pyramids ABCD and EFGH are in the same proportion as their bases. For if they were not, one of them, *e. g.* ABCD, would have a greater proportion to the pyramid EFGH, than the base BCD to the base FGH; so that a quantity less than ABCD, would have the same proportion to the pyramid EFGH, as the base BCD to the base FGH. Divide the pyramid ABCD after the manner of the third proposition; divide also the pyramids, that result from that first division, into two prisms and two pyramids, and those again into two other prisms, continuing the division as long as there shall be occasion. Since the prisms of the first division are more than the half of the pyramid ABCD, (*by the* 3.) and the prisms of the second division more than half the remainder, *i. e.* of the two lesser pyramids, and those of the third division still more than the half of what is left; it is evident, that so

ſo many diviſions may be made, that that which remains ſhall be leſs than the exceſs of the pyramid ABCD above the quantity L, that is, that all the priſms taken together ſhall be greater than the quantity L. Make as many diviſions of the pyramid EFGH, ſo that you may have as many priſms as there are in AB CD.

Demonſtration.

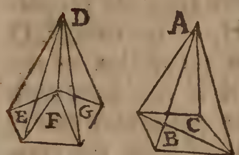
The priſms of ABCD have the ſame proportion to the priſms of EFGH, as the baſe BCD to the baſe FGH: but the proportion of the baſe BCD to the baſe FGH is the ſame with that of the quantity L to the pyramid EFGH: therefore the priſms of ABCD have the ſame proportion to the priſms of EFGH, as the quantity L to the pyramid EFGH. But alſo the priſms of ABCD are greater than the quantity L: therefore (*by the* 14. 5.) the priſms contain'd in the pyramid EFGH would be greater than the ſame pyramid EFGH, which is evidently falſe, becauſe the part cannot be greater than the whole. Therefore it muſt be granted, that no quantity leſs than one of the pyramids can have the ſame proportion to the other as the baſe to the baſe, and conſequently neither of the pyramids can have a greater proportion to the other than the baſe to the baſe.

PROP

PROPOSITION VI.

A Theorem

All ſorts of pyramids, of the ſame hight, have the ſame proportion as their baſes.



THE pyramids ABC and DEFG, of the ſame hight, are in the ſame proportion as the baſes BC and EFG. Divide the baſes into triangles.

Demonſtration.

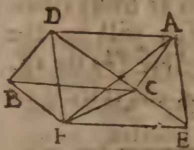
The triangular pyramids AB and DE, being of the ſame hight, are in the ſame proportion as their baſes, (*by the* 5.) So alſo the triangular pyramids AC and DF are in the ſame proportion as their baſes. Therefore the pyramid ABC has the ſame proportion to the pyramid DEF, as the baſe BC to the baſe EF, (*by the* 12. 5.) Further, ſince the pyramid DEF has the ſame proportion to the pyramid ABC, as the baſe EF to the baſe BC ; and again, the pyramid DG has the ſame proportion to the pyramid ABC, as the baſe G to the baſe BC ; the pyramid DEFG will alſo have the ſame proportion to the pyramid ABC, as the baſe EFG to the baſe BC.

PROP.

PROPOSITION VII.

A Theorem.

Every pyramid is the third part of a prism, being upon the same base, and of the same hight.



Suppose first the triangular prism AB be propos'd: I say, a pyramid, having one of the triangles ACE or BDF for its base, and being of the same hight, as the pyramid ACEF, will be the third part of the prism. Draw the three diagonals AE, DC, FC, of the three parallelograms.

Demonstration.

The prism is divided into three equal pyramids, ACFE, ACFD, and CFBD; therefore each will be the third part of the prism. The two first, having for their bases the triangles AEF and AFD, which (*by the* 34.1.) are equal, and for their hight the perpendicular drawn from the top C to the plane of their bases AF, will be equal, (*by the preceding*,) The pyramids ACFD, and CFBD, which for their bases have the equal triangles ADC and DCB, and the same top F, will be also equal, (*by the preceding.*) Therefore one of those pyramids, *e.g.* AFCE,

AFCE, having the ſame baſe BDF with the priſm, and the ſame hight, which is the perpendicular drawn from the point *F* to the plane of the baſe ACE, is the third part of the ſame priſm. If the priſm be a polygon, it muſt be divided into divers triangular priſms; and the pyramid, which has the ſame baſe, and the ſame hight, will be alſo divided into as many triangular pyramids; each of which will be the third part of its priſm. Therefore (*by the* 12. 5.) the polygon pyramid will be the third part of the polygon priſm.

PROPOSITION VIII.

A THEOREM.

Similar pyramids are in the triplicate proportion of that of their homologous ſides.

IF the pyramids be triangular, compleat the priſms, which will be alſo ſimilar, becauſe they will have certain planes the ſame with thoſe of the pyramids. But the ſimilar priſms are in the triplicate proportion of their homologous ſides; [*by Coroll.4.of the* 39.11.] therefore the pyramids, which [*by the preceding*] are the third parts of the priſms, will be in the triplicate proportion of that their homologous ſides.

If the pyramids be polygons, they muſt be reduc'd to triangular pyramids.

PROP.

PROPOSITION IX.

A THEOREM.

Equal pyramids have the hights and bases reciprocal, and those that have their hights and bases reciprocal, are equal.

IF two equal triangular pyramids be propos'd, make prisms upon the same base, and of the same hight. Since every prism is triple his pyramid, (*by the* 7.) they will also be equal. But equal prisms have their bases and hights reciprocal, (*by coroll.* 4. *of the* 39. 11.) therefore the bases and hights of the pyramids, which are the same with those of the prisms, will be also reciprocal.

Secondly, if the bases and hights of the pyramids be reciprocal, the prisms will be equal, as also the pyramids, which are the third parts of the prisms.

If the pyramids propos'd be polygons, they must be reduc'd to triangular pyramids.

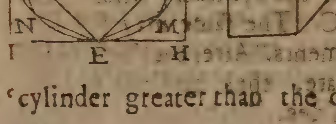
Coroll. Other propositions may be made concerning pyramids: as for example; That pyramids of the same hight, are in the same proportion as their bases; and those that have the same bases, are in the same proportion as their hights.

The USE.

'From these propositions is drawn the man-
'ner of measuring pyramids, which is, by mul-
'tiplying their bases by the third part of their
'hights. Other propositions may also be made,
'as, That if a prism be equal to a pyramid, the
'bases and the hight of the prism, with the third
'part of the hight of the pyramid, will be reci-
'procal; which is as much as to say, that if the
'base of the pyramid has the same proportion to
'the base of the prism, as the hight of the prism
'to the third part of the hight of the pyramid,
'the prism and the pyramid will be equal.

A LEMMA.

If a quantity less than a Cylinder be propos'd, a polygon prism may be inscrib'd in the Cylinder greater than that quantity.



If the quantity A be less than 'the cylinder, 'whose base is 'the circle B, a 'polygon prism 'may be in-'scrib'd in the 'cylinder greater than the quantity A. The square

'square CDEF inscrib'd in, and GHIK circum-'scrib'd about, the circle. CLDMENFO is an 'octogon inscrib'd. Draw the tangent PLQ: and 'suppose you had so many prisms as there are 'polygon bases, and all of the same hight with 'the cylinder. That which has the circumscrib'd 'square for its base, will encompass the cylin-'der; and that whose base is the inscrib'd square, 'will be also inscrib'd in the cylinder.

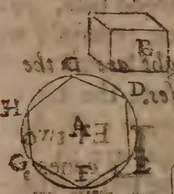
Demonstr. Prisms of the same hight are in 'the same proportion as their bases, (*by coroll.* 3. '*of the* 39. 11.) and the inscrib'd square being 'the half of that which is circumscrib'd, its 'prism will be the half of the other, and there-'fore more than the half of the cylinder. And 'making the prism with the octogon base, you 'take away more than half of what remain'd of 'the cylinder, after the prism of the inscrib'd 'square was taken from it, because the trian-'gle CLD is the half of the rectangle CQ. And 'because prisms of the same hight are in the 'same proportion as their bases, the prism, whose 'base is the triangle CLD, will be the half of 'the prism, which for its base has the rectangle 'DCPQ: it will therefore be more than the 'half of that part of the cylinder, whose base is 'the segment DLC. The same may be said of 'all the other segments. After the same manner 'I may demonstrate, that making a polygon 'prism of sixteen sides, I take away more than half

'half of what remains of the cylinder, after the 'octogon prism is taken from it: so that there 'will remain at last a part of the cylinder, less 'than the excess of the cylinder above the quan- 'tity A. We shall have therefore a prism in- 'scrib'd in the cylinder, which shall be less ex- 'ceeded by the cylinder than the quantity A, '*i.e.* which shall be greater than the quantity A. 'The same way of arguing will hold of the py- 'ramids inscrib'd in a cone.

PROPOSITION X.

A THEOREM.

A Cone is the third part of a cylinder, having the same base, and being of the same hight.



IF a cone and a cylinder have the circle A for their base, and be of the same hight, the cylinder will be triple the cone. For if the proportion of the cylinder to the cone was greater than the triple propor- tion, the quantity B less than the cylinder would have the same proportion to the cone as three to one: and (*by the preceding Lemma*) a polygon prism may be inscrib'd in the cylinder greater than the quantity B. Suppose that which has for its base the polygon CDEFGH to be such an one. Make also upon the same base a pyramid inscrib'd in the cone. De-

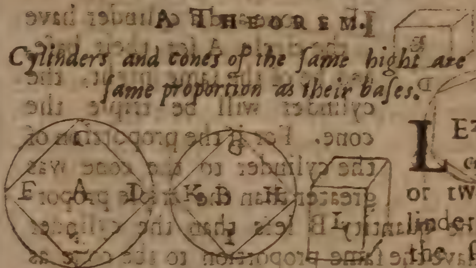
Demonstr. The cylinder, the cone, the prism, and the pyramid, are of the same hight; therefore the prism is the triple of the pyramid, (*by the 7.*) But the quantity B is also the triple of the cone; therefore the prism has the same proportion to the pyramid, as the quantity B to the cone: and (*by the* 14. 5.) the prism being greater than the quantity B, the pyramid would be greater than the cone, in which it is inscrib'd, which is impossible.

But if it be said, that the cone has a greater proportion to the cylinder than one to three, the same method may be made use of to demonstrate the contrary.

PROPOSITION XI.

A THEOREM.

Cylinders and cones of the same hight are in the same proportion as their bases.



LET two cones, or two cylinders, of the same hight, be propos'd, having for their bases the circles A and B; I say, they are in the same proportion as their bases. For if not, one of them, *e.g.* the cylinder A would have a greater proportion to the

the cylinder B, than the base A has to the base B; suppose then that the quantity L, less than the cylinder A, has the same proportion to the cylinder B, as the base A to the base B. Therefore a polygon prism may be inscrib'd in the cylinder A, which shall be greater than the quantity L. Suppose it that therefore, whose base is the polygon CDEF; and inscribe a similar polygon GHIK in the base B, which is also the base of a cylinder of the same hight.

Demonstr. The prisms of A and B are in the same proportion as their polygon bases, (*by coroll. 4. of the 39.*11.) and the polygons are in the same proportion as the circles, (*by coroll. 2. of the* 2.) therefore the prism A has the same proportion to the prism B, as the circle A to the circle B. But as the circle A to the circle B, so is the quantity L to the cylinder B: therefore as the prism A to the prism B, so is the quantity L to the cylinder B. But the prism A is greater than the quantity L, and consequently [*by the* 14.5.] the prism B, inscrib'd in the cylinder B, would be greater than its cylinder, which is impossible. Therefore neither of the cylinders has a greater proportion to the other, than its base to the other's base.

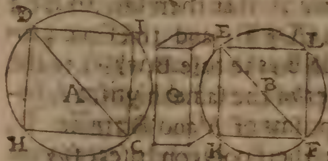
Coroll. Cylinders are triple the cones, of the same hight, therefore cones of the same hight are in the same proportion as their bases.

PROP.

PROPOSITION XII.

A Theorem.

Cylinders and cones, that are similar, are in the triplicate proportion of that of the diameters of their bases.



Let two cones or two cylinders, that are similar, be propos'd, having the circles A and B for their bases; I say, that the proportion of the cylinder A to the cylinder B is the triplicate proportion of that of the diameter DC to the diameter EF. For if it be not the triplicate proportion; let the quantity G, less than the cylinder A, be, to the cylinder B, in the triplicate proportion of that of the diameter DC to the diameter EF; and inscribe a prism in the cylinder A greater than G, and another similar to it, in the cylinder B: they will be of the same hight with the cylinder, because similar cylinders have their hights and the diameters of their bases proportional, as well as prisms, (*by defin.* 22. 11.)

Demonstr. The diameter DC has the same proportion to the diameter EF as the side DI to the side EL, or as DC to EF, *as I have shown in*

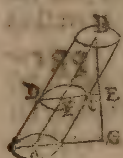
in the first. But similar prisms are in the triplicate proportion of that of their homologous sides, (*by coroll. 4. of 39. 11.*) therefore the prism A to the prism B is in the triplicate proportion of that of DC to EF. But we suppos'd that the quantity G in respect of the cylinder B was in the triplicate proportion of that of DC to EF; therefore the prism A will have the same proportion to the prism B as the quantity G to the cylinder B; and (*by the 14. 5.*) the prism A being greater than the quantity G, the prism B, inscrib'd in the cylinder B, will be greater than the cylinder B, which is impossible. Therefore similar cylinders are in the triplicate proportion of that of the diameters of their bases.

Cones are the third parts of Cylinders, [*by the 10.*] therefore similar cones are in the triplicate proportion of that of the diameters of their bases.

PROPOSITION XIII.

A THEOREM.

If a cylinder be cut by a plane, that is parallel to to its base, the parts of its axis will be in the same proportion as the parts of the cylinder.



LET the cylinder AB be cut by the plane DC parallel to its base: I say, the cylinder AF will have the same proportion to the cylinder FB, as the line AF to the line

line FB. Draw the line BG perpendicular to the plane of the base A. Draw also upon the planes of the circles DC and A the lines FE and AG. *Demonstration.*

The plane of the triangle BAG cuts the parallel planes A and DC; therefore the sections *FE* and AG are parallel, (*by the 16.11.*) So that *AF* has the same proportion to *FB*, as the hight GE to EB. Take any aliquot part of EB; and having divided GE and EB into parts equal to it, draw so many planes parallel to the base A; then will you have so many cylinders of the same hight; which, having their bases and hights equal, will be equal, (*by the 11.*)

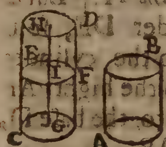
Further, the lines *AF* and *FB* will be divided after the same manner as EG and EB, [*by the* 17.11.] so that the line AF will as oft contain any aliquot part of the line *FB*, as the cylinder AP contains the like aliquot part of the cylinder *FB*; therefore the parts of the cylinder will be in the same proportion as the parts of their axis.

Corol. The parts of the perpendicular are in the same proportion as the parts of the cylinder.

PROPOSITION XIV.

A THEOREM.

Cylinders and cones, having the same bases, are in the same proportion as their hights.



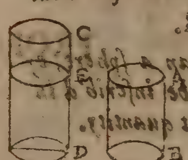
TWO cylinders of equal bases, being propos'd, as AB and CD, cut in the greater a cylinder of the same hight with the less, drawing a plane EF parallel to its base. 'Tis evident [*by the 11.*] that the cylinders *CF* and *AB* are equal; and that CF has the same proportion to CD, as GE to GH, or [*by the coroll. of the preceding*] as the hight of CF to the hight of CD; therefore AB has the same proportion to CD, as the hight of CF or AB, to the hight of CD.

Cones, being the third parts of cylinders, if their bases be equal, will be also in the same proportion as their hights.

PROPOSITION XV.

A THEOREM.

Cylinders and cones that are equal, have their bases and hights reciprocal: and those, that have their bases and hights reciprocal, are equal.



IF the cylinders AB and CD be equal, the base B will have the same proportion to the base D, as the hight CD

to the hight AB. Take the hight DE equal to the hight AB.

Demonstr. The cylinder AB has the same proportion to the cylinder DE, of the same hight, as the base B to the base D, [*by the* 11.] But as the cylinder AB is to the Cylinder DE, so is the cylinder CD, equal to AB, to the cylinder DE; *i.e.* so is the hight CD to the hight AB or DE. Therefore as the base B to the base D, so is the hight CD to the hight AB.

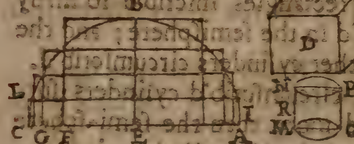
Secondly, if the base B has the same proportion to the base D, as the hight CD to the hight AB, the cylinders AB and CD will be equal. For the cylinder AB is in the same proportion to the cylinder DE, as the base B to the base D; and the cylinder CD will have the same proportion to DE, as the hight CD to the hight DE: therefore AB has the same proportion to DE, as CD to DE; and [*by the* 9. 5.] the cylinders AB and CD will be equal.

"The 16. and 17. Propositions are very difficult and of no other use but to prove the 18. "which may more easily be done by the following Lemma's.

LEMMA I.

If a quantity be propos'd less than a sphere, cylinders of the same hight may be inscrib'd in the same sphere greater than that quantity.

Sup-



‘Suppose ABC to be a great semicircle of the sphere, whereof we treat, and the quantity D to be the quantity less than that sphere: I say, several cylinders of the same hight may be inscrib'd in the sphere, which taken together will be greater than the quantity D. For if the semi-sphere exceed the quantity D, it will exceed it by some magnitude; let it then be the cylinder MP, so that the quantities D and MP taken together may be equal to the semi-sphere. Make a great circle of the sphere to have the same proportion to the base MO, as the hight MN to the hight R. Then divide the line EB into as many equal parts as you please, each being less than R: and drawing parallels to the line AG, describe the inscrib'd and circumscrib'd parallelograms. The number of the circumscrib'd will exceed that of the inscrib'd by one. But all the rectangles circumscrib'd will surpass all the inscrib'd by the little rectangles through which the circumference of the circle passes; all which taken together are equal to the rectangle AL. I imagine then the semicircle to be turn'd about upon the diameter EB; the semicircle will by that motion describe a semi-sphere,

'sphere, and the rectangles inscrib'd so many
'cylinders inscrib'd in the semisphere; and the
'circumscrib'd, other cylinders circumscrib'd.

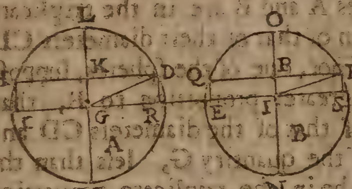
Demonstr. The circumscrib'd cylinders sur-
'pass the inscrib'd more than the semi-sphere
'surpasses the same inscrib'd cylinders, it being
'contain'd within the circumscrib'd cylinders.
'But the circumscrib'd surpass the inscrib'd by
'the cylinder AL; therefore the semisphere
'will surpass those inscrib'd cylinders by less
'than the cylinder describ'd by the rectangle
'AL. But the cylinder AL is less than the cy-
'linder MP; for there is the same proportion
'of a great circle of the sphere, which is the
'base of the cylinder AL, to MO, as of MN to
'R; therefore (*by the preceding*) a cylinder,
'which should have a great circle of the sphere
'for its base, and the hight R, would be equal
'to the cylinder MP; but the cylinder AL, tho
'it have the same base, yet it's hight CL is less
'than R; therefore the cylinder AL is less than
'the cylinder MP. Consequently the semi-
'sphere, that exceeds the quantity D by the cy-
'linder MP, and the inscrib'd cylinders by a
'quantity less than AL, exceeds the inscrib'd
'cylinders by less, than it exceeds the quantity
'D; therefore the quantity D is less than the
'cylinders inscrib'd in the semisphere.

'That which I have said of the semi-sphere, is
'applicable to an entire sphere.

LEM-

LEMMA II.

Similar cylinders, inscrib'd in two spheres, are in the triplicate proportion of the diameters of the spheres.



IF two similar cylinders CD and EF be inscrib'd in the spheres A and B, they will be in the triplicate proportion of the diameters LM and NO. Draw the lines GD and IF.

Demonstration.

The right cylinders CD and EF are similar; therefore HD has the same proportion to DR as QF to FS, as also KD has the same proportion to DG as PF to FI. Consequently the triangles GDK and IFP are similar, (*by the 6, 6.*) therefore KD has the same proportion to to *PF* as GD to *IF*, or LM to ON. But the similar cylinders CD and EF are in the triplicate proportion of KD and *PF*, the semidiameters of their bases, (*by the 12.*) therefore the similar cylinders CD and EF, inscrib'd in the spheres A and B, are in the triplicate proportion of the diameters of the spheres.

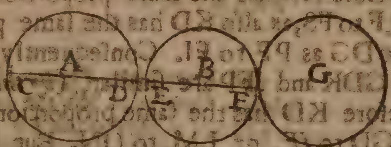
PROP.

PROPOSITION XVIII.

A THEOREM.

Spheres are in the triplicate proportion of their diameters.

THE spheres A and B are in the triplicate proportion of that of their diameters CD and EF. For if not, one of the spheres, suppose A, will be in a greater proportion to B, than the triplicate of that of the diameters CD and EF; therefore the quantity G, less than the sphere A, will be in the triplicate proportion of that of CD to EF, to the sphere B; and then some cylinders may (*according to the Lem.* 1.) be inscrib'd in the sphere A, greater than the quantity G. Inscribe an equal number of cylinders in the sphere B, similar to those in the sphere A.



Demonstr. The cylinders of the sphere A to those of the sphere B are in the triplicate proportion of that of CD to EF, but the quantity G to the sphere B is also in the triplicate proportion of that of CD to EF: therefore the cylinders of the sphere A have the same proportion to the similar cylinders of the sphere B, as the

the quantity G to the ſphere B. Conſequently the cylinders of A being greater than the quantity G, the cylinders of B, *i. e.* inſcrib'd in the ſphere B, will be greater than the ſphere B, which is impoſſible. Therefore the ſpheres A and B are in the triplicate proportion of that of their diameters.

Coroll. Spheres are in the ſame proportion as the cubes of their diameters; becauſe cubes, being ſimilar ſolids, are in the triplicate proportion of their ſides, [*by the* 33.11.]

The END.

ERRATA.

Pag. 1. line 12. read *Propoſitions.* Pag. 10. l. 8. add *An.* p. 24. l. ult. dele *be.* Prop. 22. 1 Sch. the letters *A* and *B* are Tranſpoſed. p.64 Sch. 2. add F to line *EB* produced below *CD.* p. 78, l. 15 r. *not.* l. 23. r. by the 37. pag. 99. l. penult. dele *of.* p. 101. l. 2. for *EC* read *FB.* pag. 151. l. 6. for *AE* r. *AI.* p. 167. l. 3. for *EF* r. *DF.* p. 180. l. 10. for *EH* r. *FH.* p. 199. l. 20. del. *of* p. 209. l. 9. for *had* read *have* p. 260. l. 13. for *AC* r. *EC.* l. 17. for *AB* r. *DB.* p. 263. ſch. transfer *A* to the other ſide of the ſcheme.